Complete calculation of electroweak corrections to the Drell–Yan process for LHC

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Introduction

Despite the fact that the Standard Model (SM) more than twenty years keeps for oneself the status of consistent and experimentally confirmed theory, the search of New Physics (NP) manifestations is continued. The possible traces of NP can be the supersymmetry, the large extra dimensions, extra neutral gauge bosons, compositness of fermions, the anomalies in vertices and so on. Forthcoming experiments at the LHC probably will shed the light on these problems.

One of powerful tool in the experiments at LHC from the point of view for the NP exploration is the experimental investigation of the continuum for the Drell-Yan production of dilepton pair, i.e. data on the cross section and the forward-backward asymmetry of the reaction

$$pp \to \gamma, Z \to l^+ l^- X \tag{1}$$

at large invariant mass of dilepton pair ($M \geq 1$ TeV).

The studies of NP effects is impossible without the exact knowledge of SM predictions including higher order ElectroWeak radiative Corrections (EWC):

- (I) EWC induced by gauge Boson Self-Energies (BSE),
- (II) the others QED corrections (i.e. radiative corrections induced by at least one additional photon: virtual or real),
- (III) the others Weak corrections (i.e. radiative corrections induced by additional heavy bosons: Z or W),

The (I) and (II) are studied well (see papers on pure QED corrections:
V. Mosolov and N. Shumeiko, Nucl.Phus. B **186**, 394 (1981),
A. Soroko and N. Shumeiko, Yad. Fiz **52**, 514 (1990))
as well as QED corrections and EWC in the Z peak region and above in paper
U. Baur et al., Phys. Rev. D **65**: 033007, (2002)
and calculation of SANC project:
A. Andonov, A. Arbuzov, D. Bardin et al., Comput. Phys. Commun. **174**, 481 (2006)

Both accurate and fast!

The important task is the insertion of this background into the LHC Monte Carlo generators and they should be both accurate and fast. For the latter it is necessary to have the set of compact formulas for the EWC.

Mathematical contents

To get leading effect in the region of large invariant dilepton mass we actively used the so-called Sudakov logarithms (SL) (V. Sudakov, Sov. Phys. JETP **3**, 65 (1956)),

$$l_{i,x} = \log \frac{m_i^2}{|x|} \quad (i = Z, W; \;\; x = s, t, u). \tag{2}$$

Collinear logarithms play leading role in QED-part

$$\log \frac{m_l^2}{|x|} \quad (l = e, \mu; \;\; x = s, t, u). \tag{3}$$

Common convolution formula fot V-contribution

$$\sigma_V^H = \frac{1}{3}\int\limits_0^1 dx_1 \int\limits_0^1 dx_2 \int\limits_{-S}^0 dt \sum_{q=u,d,s,c,b} [f_q^A(x_1,Q^2) f_{\bar{q}}^B(x_2,Q^2)\hat{\sigma}_V^{q\bar{q}}(t) +$$

$$+ f_{\bar{q}}^A(x_1,Q^2) f_q^B(x_2,Q^2)\hat{\sigma}_V^{\bar{q}q}(t)]\theta(t+\hat{s})\theta_M\hat{\theta}_D, \qquad (4)$$

where $V = 0, \text{BSE}, \text{HV}, b, fin$; $b = \gamma\gamma, \gamma Z, ZZ, WW$.

$$\theta_M = \theta(\hat{s} - M_1^2)\theta(M_2^2 - \hat{s}) \qquad (5)$$

$$\theta_D = \theta(\zeta^* - \cos\theta)\theta(\zeta^* + \cos\theta)\theta(\zeta^* - \cos\alpha)\theta(\zeta^* + \cos\alpha)$$

$$\theta(p_T(l^+) - p_T^{min})\theta(p_T(l^-) - p_T^{min}) \qquad (6)$$

We use the standard set of Mandelstam invariants for the partonic elastic scattering $s,\ t,\ u$:

$$s = (p_1 + p_2)^2,\ t = (p_1 - k_1)^2,\ u = (k_1 - p_2)^2, \tag{7}$$

hadronic invariant $S = (P_A + P_B)^2$ and invariant mass of a dilepton $M = \sqrt{(k_1 + k_2)^2}$. The propagator for j-boson has the form

$$D^{js} = \frac{1}{s - m_j^2 + i m_j \Gamma_j}, \tag{8}$$

where m_j (Γ_j) is the j-boson mass (width).

Notations and Born cross section

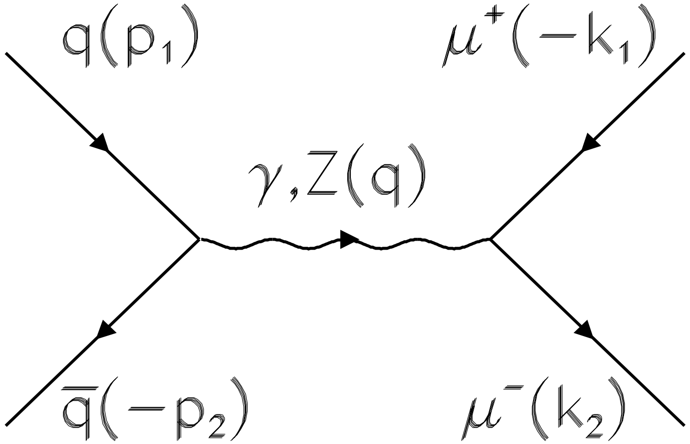


Figure 1: **The lowest order graph giving contribution to the DY scattering at parton level**

V-contributions: BORN

Born:

$$\sigma_0^{q\bar{q}}(t) = \frac{2\pi\alpha^2}{s^2} \sum_{i,j=\gamma,Z} D^i D^{j*} (b_+^{i,j} t^2 + b_-^{i,j} u^2), \qquad (9)$$

$$b_\pm^{n,k} = \lambda_{q+}^{n,k} \lambda_{l+}^{n,k} \pm \lambda_{q-}^{n,k} \lambda_{l-}^{n,k}, \qquad (10)$$

$$\lambda_{f+}^{i,j} = v_f^i v_f^j + a_f^i a_f^j, \; \lambda_{f-}^{i,j} = v_f^i a_f^j + a_f^i v_f^j, \qquad (11)$$

$$v_f^\gamma = -Q_f, \; a_f^\gamma = 0, \; v_f^Z = \frac{I_f^3 - 2s_W^2 Q_f}{2s_W c_W}, \; a_f^Z = \frac{I_f^3}{2s_W c_W}. \qquad (12)$$

All formulas can be found in
V. Zykunov, Yad. Fiz. **69**, 1557 (2006)
V. Zykunov, Phys. Rev. D **75**, 073019 (2007)

Boson Self Energies

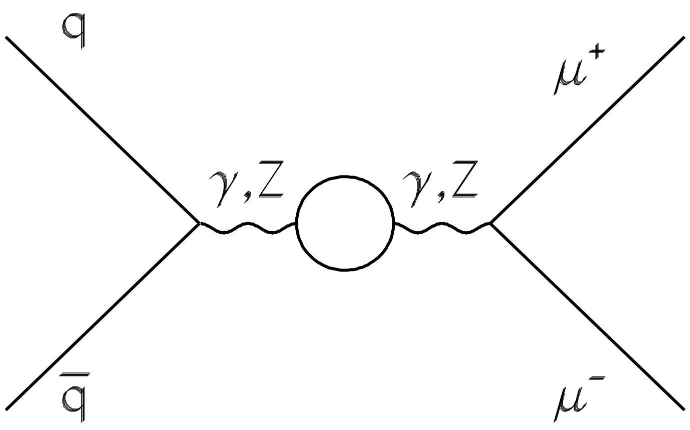


Figure 2: $\gamma\gamma$ – , γ Z – and ZZ – Self Energy diagrams

$$\sigma^{q\bar{q}}_{\mathrm{BSE}}(t) = -\frac{4\alpha^2\pi}{s^2}\Big[\sum_{i,j=\gamma,Z}\Pi^i_S D^i D^{j*}\sum_{\chi=+,-}\lambda_{q_\chi}^{i,j}\lambda_{l_\chi}^{i,j}B_\chi + $$

$$+\Pi_S^{\gamma Z} D^Z \sum_{i=\gamma,Z} D^{j*}\sum_{\chi=+,-}(\lambda_{q_\chi}^{\gamma,j}\lambda_{l_\chi}^{Z,j}+\lambda_{q_\chi}^{Z,j}\lambda_{l_\chi}^{\gamma,j})B_\chi\Big]. \quad (13)$$

$\Pi^{\gamma,Z,\gamma Z}$ are connected with the renormalized γ–, Z– and γZ–self energies as

$$\Pi^\gamma = \frac{\hat{\Sigma}^\gamma}{s},\ \Pi^Z = \frac{\hat{\Sigma}^Z}{s-m_Z^2},\ \Pi^{\gamma Z} = \frac{\hat{\Sigma}^{\gamma Z}}{s}.$$

Light and Heavy Vertices (LV and HV)

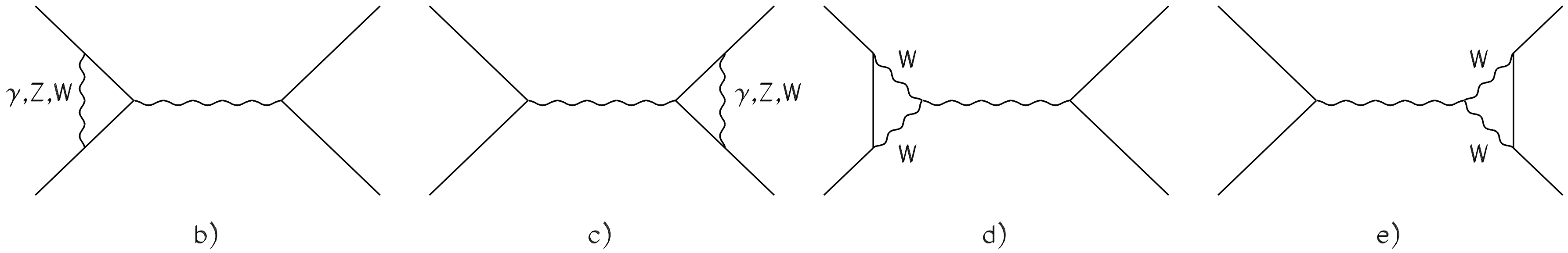


Figure 3: **Feynman graphs for Vertieces diagrams. Unsigned helix lines mean γ or Z.**

Main features of our calculation:

- the t'Hooft-Feynman gauge,
- on-mass renormalization scheme ($\alpha, m_W, m_Z,\ m_H$ and the fermion masses are independent parameters),
- ultrarelativistic limit.

The results are presented as the form factor set to the Born vertices (as, for example, in (M. Böhm *et al.*, Fortschr. Phys. **34**, 687 (1986)), so we can easily use they to construct the cross section: all that we need is to replace the coupling constants in Born vertex to the corresponding form factors:

$$v_f^j \to \delta F_V^{jf}, a_f^j \to \delta F_A^{jf}. \tag{14}$$

Electroweak form factors $\delta F_{V,A}^{if}$ in ultrarelativistic limit have a form, for example:

$$\delta F_V^{\gamma l} = \frac{\alpha v_l^\gamma}{4\pi}[((v_l^Z)^2 + (a_l^Z)^2)\Lambda_2(m_Z) + \frac{3}{4s_W^2}\Lambda_3(m_W)] \tag{15}$$

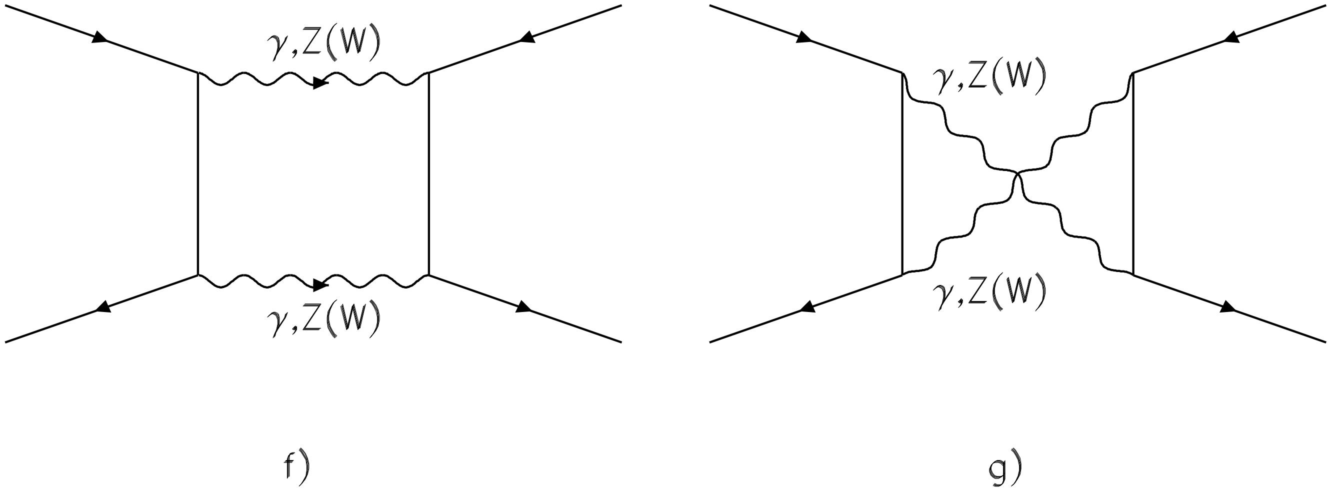
and so on... Here we give the real part of expressions for functions $\Lambda_{2,3}(m_i)$ through the Sudakov logarithms:

$$\Lambda_2(m_i) = \frac{\pi^2}{3} - \frac{7}{2} - 3l_{i,s} - l_{i,s}^2, \quad \Lambda_3(m_i) = \frac{5}{6} - \frac{1}{3}l_{i,s}. \tag{16}$$

V-contributions: LV and HV

$$\sigma^{q\bar{q}}_{\rm HV}(t) = \frac{4\pi\alpha^2}{s^2}{\rm Re}\sum_{i,j=\gamma,Z} D^i D^{j*} \sum_{\chi=+,-} (\lambda^{{\rm F}\,i,j}_{q\ \chi}\lambda^{\ i,j}_{l\chi} + \lambda^{\ i,j}_{q\chi}\lambda^{{\rm F}\,i,j}_{l\ \chi})B_\chi. \qquad (17)$$

Light and Heavy Boxes



f)

g)

Figure 4: **Feynman graphs for Boxes.**

The calculation of two heavy boson contribution is more complicate procedure since it demands the integration of 4-point functions with complex masses in unlimited from above kinematical region of invariants (see pioneer paper: G.’t Hooft and M. Veltman, Nucl. Phys. B **153**, 365 (1979)). Fortunately there is a way to avoid many of troubles with the integration all of terms in box contribution - Asymptotic Approach (AA).

First of all we construct the box cross section for $q\bar{q} \to l^+l^-$ using the standard Feynman rules:

$$d\sigma_{ZZ} = -\frac{4\alpha^3}{\pi s}\delta(q-p_1-p_2)\frac{d^3k_1}{2k_1^0}\frac{d^3k_2}{2k_2^0}\mathrm{Re}\frac{i}{(2\pi)^2}\int d^4k \sum_{k=\gamma,Z} D^{ks^*}(D^{ZZ}+C^{ZZ}), \tag{18}$$

here $D^{ZZ}(C^{ZZ})$ is contribution of direct (crossed) diagram.

Neglecting of fermion masses and polarization of particles we present the direct contribution in the form:

$$D^{ZZ} = \frac{\mathrm{Tr}[\gamma^\alpha \hat{p}_2\gamma_\mu(\hat{p}_1-\hat{k})\gamma_\nu \rho_q^{ZZ,k}(p_1)]\mathrm{Tr}[\gamma_\alpha \hat{k}_2\gamma^\mu(\hat{k}-\hat{k}_1)\gamma^\nu \rho_l^{ZZ,k}(k_1)]}{((q-k)^2-m_Z^2)(k^2-m_Z^2)(k^2-2k_1k)(k^2-2p_1k)}, \tag{19}$$

Combinations of the density matrices $\rho(p)$ and the coupling constants can be reduced to production of λ-factors (as in the "Born" formulas). For the crossed part:

$$C^{ZZ} = -D^{ZZ}\Big|^{b_+^{ZZ,k}\leftrightarrow b_-^{ZZ,k}}_{t\leftrightarrow u},\ b_\pm^{n,k} = \lambda_{q_+}^{n,k}\lambda_{l_+}^{n,k} \pm \lambda_{q_-}^{n,k}\lambda_{l_-}^{n,k}. \tag{20}$$

To extract the part of cross section which predominates in region $s,\ |t|,\ |u| \gg m_Z^2$ we should make equivalent transformation based on the close connection of infrared divergency and SL terms:

$$D^{ZZ} = (D^{ZZ}_{k\to 0} + D^{ZZ}_{k\to q}) + (D^{ZZ} - D^{ZZ}_{k\to 0} - D^{ZZ}_{k\to q}) = D^{ZZ}_1 + D^{ZZ}_2. \qquad (21)$$

Integrating over k and retaining the terms which are proportional to the second ($\sim\ l^2_{i,x}$), first ($\sim\ l^1_{i,x}$) and zero ($\sim\ l^0_{i,x}$) power of Sudakov logarithms we get the asymptotic expressions

$$\frac{i}{(2\pi)^2}\int d^4kD^{ZZ}_1 \approx -\frac{2}{s}(b^{ZZ,k}_{+}t^2+b^{ZZ,k}_{-}u^2)(\frac{\pi^2}{3}+\frac{1}{2}l^2_{Z,t}),\ \text{t'Hooft and Veltman, 79} \qquad (22)$$

$$\frac{i}{(2\pi)^2}\int d^4kD^{ZZ}_2 \approx b^{ZZ,k}_{-}u\log\frac{s}{|t|}+(b^{ZZ,k}_{-}(t^2+u^2)+2b^{ZZ,k}_{+}t^2)\frac{1}{2s}\log^2\frac{s}{|t|},\ \text{Kahane, 64} \qquad (23)$$

Masses of fermions and UV parameter L are cancelled out.

Retaining only leading $\sim\ l^2_{i,x}$ term how it had been done in P. Ciafaloni and D. Comelli, Phys. Lett. B **446**, 278 (1999) we get coincidence with the results of this paper.

To obtain the WW-box contribution to Drell-Yan cross section one should:
1) to do the trivial substitution in all of indices of coupling constants and boson masses $Z \to W$,
2) to take into consideration that some parton diagrams are forbidden by the charge conservation law (direct WW-box: $d\bar{d} \to l^+l^-$ and $\bar{u}u \to l^+l^-$; crossed WW-box: $u\bar{u} \to l^+l^-$ and $\bar{d}d \to l^+l^-$). This second feature of WW-boxes explains the fact of domination of WW-contribution to Drell-Yan cross section in comparison with ZZ (or γZ) -contribution (see below in numerical analysis). Point is that the leading term of ZZ-contribution is proportional to difference

$$\delta^{ZZ,k}(t,u,b_+,b_-) - \delta^{ZZ,k}(u,t,b_-,b_+) \sim l^2_{Z,t} - l^2_{Z,u} = \log\frac{u}{t}(l^1_{Z,t} + l^1_{Z,u}), \quad (24)$$

i.e. leading terms of ZZ-box contribution $\sim l^1_{Z,x}$, whereas the leading parts of WW–cross section do not contain the difference (24) and are proportional to $l^2_{W,x}$. Let us remark here that the factorization property (24) is absent in heavy vertex part and takes place for infrared finite part of γZ–box contribution.

Boxes:

$$\sigma_b^{q\bar{q}}(t) = \frac{2\alpha^3}{s^2} \sum_{k=\gamma,Z} D^{k^*}[\delta^{b,k}(t,u,b_+,b_-) - \delta^{b,k}(u,t,b_-,b_+)]. \tag{25}$$

Fin-part:

$$\sigma_{fin}^{q\bar{q}}(t) = \frac{\alpha}{\pi}\delta_{fin}^{q\bar{q}}\sigma_0^{q\bar{q}}(t), \tag{26}$$

$$\begin{aligned} \delta_{fin}^{q\bar{q}} = \quad & J_0 \log\frac{2\omega}{\sqrt{s}} + Q_l^2(3\log\frac{\sqrt{s}}{m} - 2 + \frac{\pi^2}{3}) + Q_q^2(3\log\frac{\sqrt{s}}{m_q} - 2 + \frac{\pi^2}{3}) \\ & -Q_q Q_l(\log\frac{s^2}{tu}\log\frac{t}{u} + \frac{\pi^2}{3} + \log^2\frac{t}{u} + 4\mathrm{Li}_2\frac{-t}{u}), \end{aligned} \tag{27}$$

$$\frac{1}{2}J_0 = Q_q^2\Big(\log\frac{s}{m_q^2} - 1\Big) - 2Q_q Q_l \log\frac{t}{u} + Q_l^2\Big(\log\frac{s}{m^2} - 1\Big). \tag{28}$$

Comparison with existing results: BSE

$\sqrt{s}$, TeV	BSE, SANC	BSE, our
0.1	6.0777	8.1119
0.2	11.2259	12.2144
0.5	11.1526	11.9455
1.0	12.2096	12.9793
2.0	13.1993	13.9634
3.0	13.7682	14.5314
5.0	14.4811	15.2437
10.0	15.4456	16.2080

Table 1. The relative corrections (in per cents) from the BSE– cross section at the parton level for $u\bar{u} \to \mu^+\mu^-$ as a functions of $\sqrt{s}$, calculated by different groups: SANC and our calculation.

Comparison with existing results: HV

$\sqrt{s}$, TeV	HV, SANC	HV, exact	HV, AA
0.2	-1.6883	-1.6688	2.4481
0.5	4.2958	4.2978	4.0943
1.0	6.2447	6.2451	5.9910
2.0	8.5534	8.5535	8.4247
3.0	10.1709	10.1710	10.0935
5.0	12.4894	12.4895	12.4512
10.0	16.1160	16.1160	16.1024

Table 2. The relative corrections (in per cents) from the $\mathrm{HV}-$ cross section at the parton level for $u\bar{u} \to \mu^+\mu^-$ as a functions of $\sqrt{s}$, calculated by SANC and using these results: exact and AA.

Comparison with existing results: ZZ and WW

$\sqrt{s}$, TeV	ZZ, SANC	ZZ, ZGRAD	ZZ, AA	WW, SANC	WW, AA
0.1	-0.0186		0.0683	-0.329	-1.2690
0.2	-0.0908	-0.0907	-0.0073	-3.107	-4.8739
0.5	-0.2144	-0.2145	-0.1895	-10.777	-10.1087
1.0	-0.3346	-0.3346	-0.3251	-16.998	-16.5720
2.0	-0.4638	-0.465	-0.4612	-25.442	-25.2497
3.0	-0.5423	-0.543	-0.5410	-31.468	-31.3554
5.0	-0.6432	-0.643	-0.6416	-40.185	-40.1306
10.0	-0.7787	-0.779	-0.7782	-53.989	-53.9695

Table 3. The relative corrections (in per cents) from the ZZ- and WW-box cross section at the parton level for $u\bar{u} \to \mu^+\mu^-$ as a functions of $\sqrt{s}$, calculated by different groups: SANC, program ZGRAD (U. Baur et al. `http://ubhex.physics.buffalo.edu/~baur/zgrad2.tar.gz`) and using the asymptotic formulas (AA).

Hard bremsstrahlung

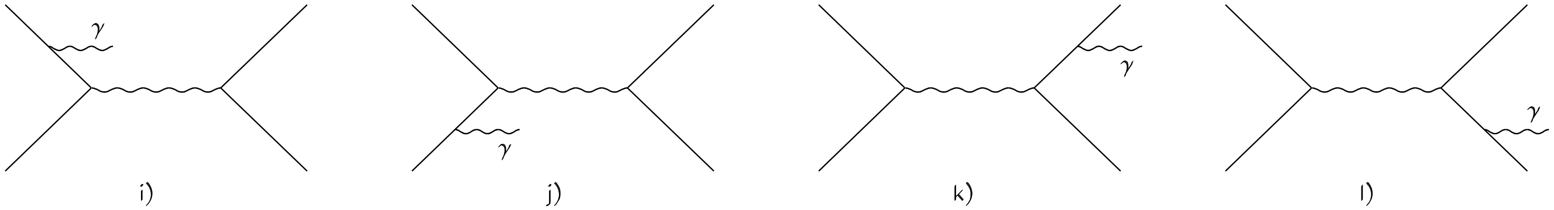


Figure 5: Bremsstrahlung diagrams. Unsigned helix lines mean γ or Z.

$$I^6_\Omega[A] = \int\limits_0^1 dx_1 \int\limits_0^1 dx_2 \iiiint\limits_\Omega dt dv dz du_1 \frac{1}{\pi\sqrt{R_{u_1}}}\theta(\hat{R}_{u_1})\theta^R_M\hat{\theta}^R_D\; A, \qquad (29)$$

with radiative invariants $z = 2k_1p,\ v = 2k_2p,\ z_1 = 2p_1p,\ u_1 = 2p_2p$, and p – 4-momenta of real photon.

For numerical integration we used Monte Carlo routine based on the VEGAS algorithm (G. Peter Lepage, J. Comput.Phys. **27**, 192 (1978))

Discussion of numerical results. Code READY.

In the following the scale of radiative corrections and their effect on the observables of the Drell-Yan processes will be discussed using FORTRAN program READY (V. Zykunov, hep-ph/0702203V2).

READY is "Radiative corrEctions to lArge invariant mass Drell-Yan process".

We used the following set of prescriptions:

- investigated reaction is (1) with the energy of LHC $\sqrt{S} = 14$ TeV,
- the set of SM input electroweak parameters: $\alpha = 1/137.03599911$, $m_Z = 91.1876$ GeV, $m_W = 80.37399$ GeV, $\Gamma_Z = 2.4924$ GeV, $\Gamma_W = 2.0836$ GeV, $m_H = 115$ GeV,
- muon mass $m_\mu = 0.105658369$ GeV, masses of fermions for loop contributions to the BSE: $m_e = 0.51099892$ keV, $m_\tau = 1.77699$ GeV, $m_u = 0.06983$ GeV, $m_c = 1.2$ GeV, $m_t = 174$ GeV, $m_d = 0.06984$ GeV, $m_s = 0.15$ GeV, $m_b = 4.6$ GeV (the light quark masses provide $\Delta\alpha^{(5)}_{had}(m_Z^2)$=0.0276),

- 5 active flavors of quarks in proton, their masses as regulators of the collinear singularity $m_q = 10 \times m_u$,

- non-diagonal elements of CKM matrix = 0, diagonal ones = 1,

- "soft"-"hard" photon separator $\omega = 10$ GeV,

- the MRST2004QED set of unpolarized parton distribution functions (with the choice $Q = M_{sc} = m_Z$),

- we impose the experimental restriction conditions on the detected lepton angle $-\zeta^* \le \zeta \le \zeta^*$ and on the rapidity $|y(l)| \le y(l)^*$, see (6); for CMS detector the cut values of ζ^* and $y(l)^*$ are determined as

  $$y(l)^* = -\log\ \tan\frac{\theta^*}{2} = 2.5,\ \ \zeta^* = \cos\theta^* \approx 0.986614, \tag{30}$$

  also we used the second standard CMS restriction $p_T(l) \ge 20$ GeV,

- here we used so-called "bare" setup for muons identification requirements (no smearing, no recombination of muon and photon).

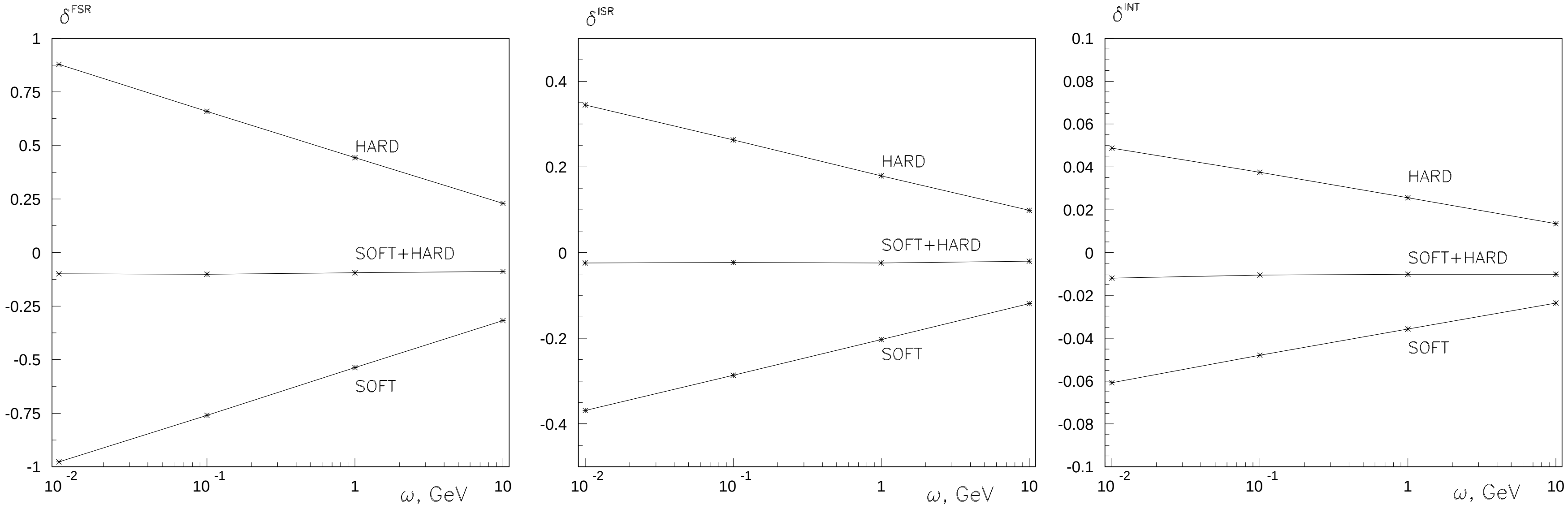


Figure 6: Independence of the FSR, ISR and INT-parts of bremsstrahlung cross section on separator ω.

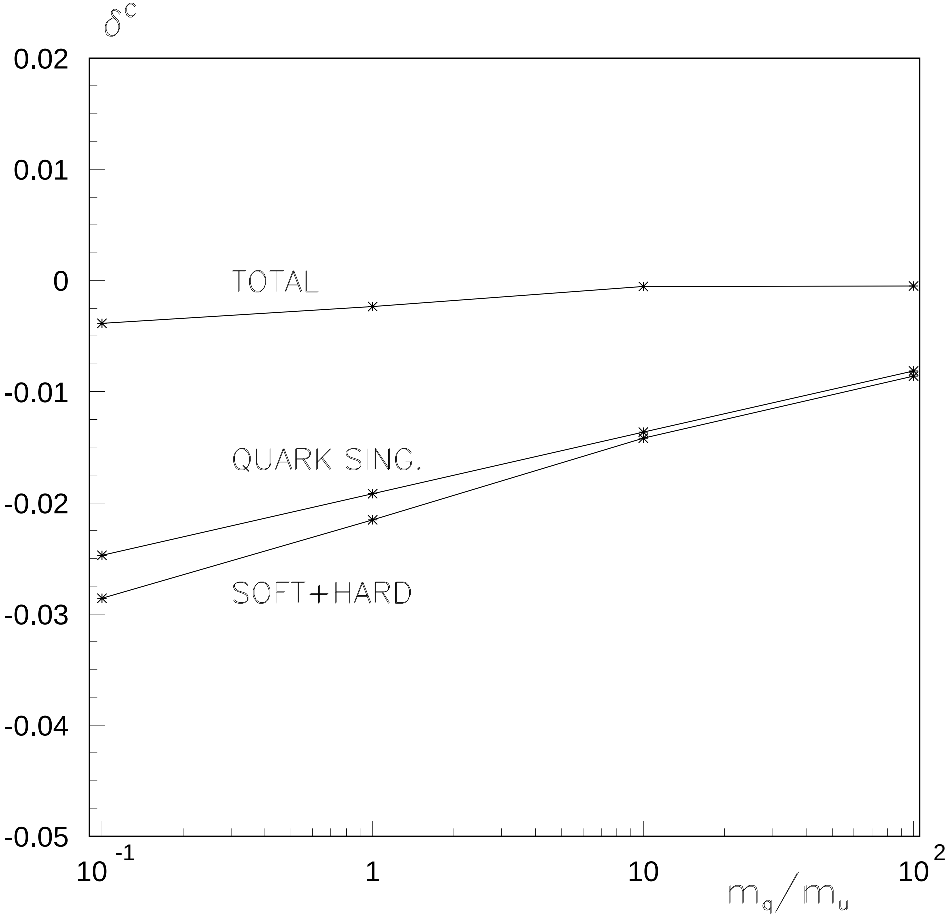


Figure 7: Independence of the ISR-part of bremsstrahlung cross section on quark mass m_q, the "TOTAL" means "SOFT+HARD-QUARK SING.", we suppose here $\omega = 10$ GeV.

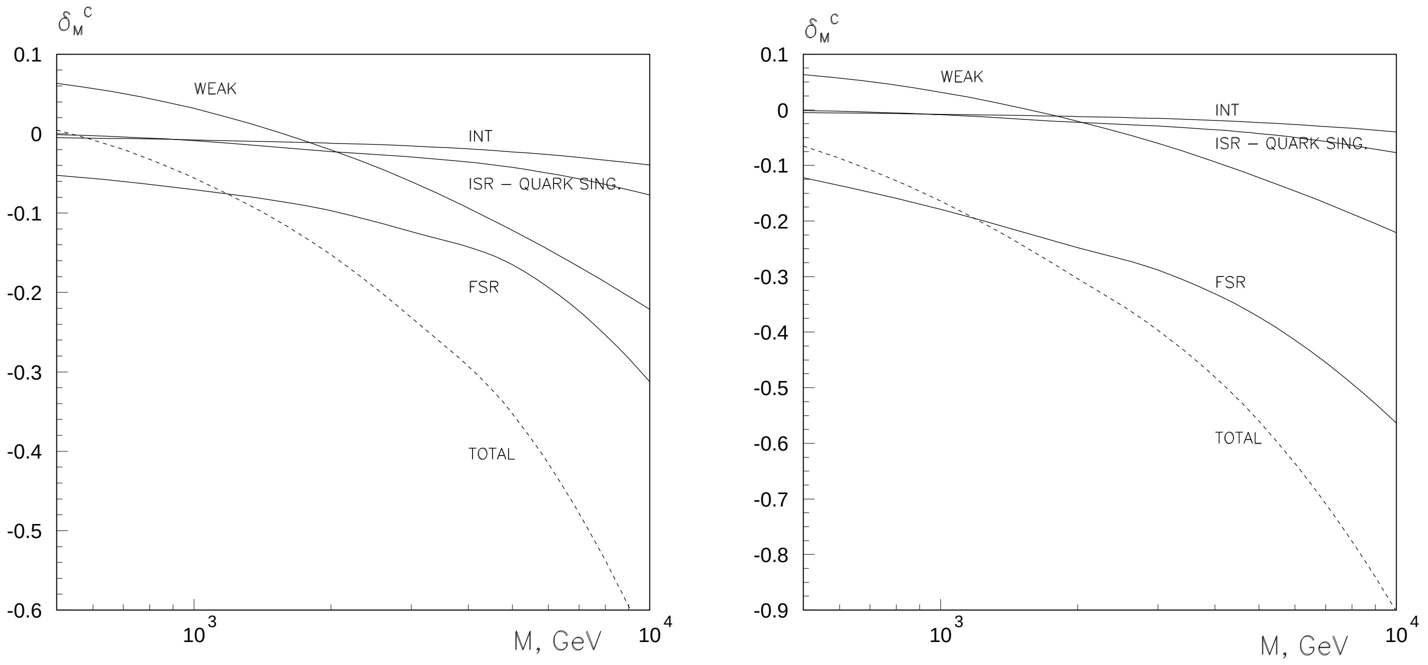


Figure 8: The relative corrections δ_M^C corresponding to different contributions (total corrections is denoted by dashed line) with experimental restrictions of the CMS as a functions of M. Left picture for $l = \mu$, right one for $l = e$.

Rel. corrections are $\delta_M^C = \frac{\sigma_C(M)}{\sigma_0(M)}$. Expected relative experimental precision from CMS ($100\ \mathrm{fb}^{-1}$, M=1 TeV) is $\sim$ 2% (CERN/CMS Note 2000-035)

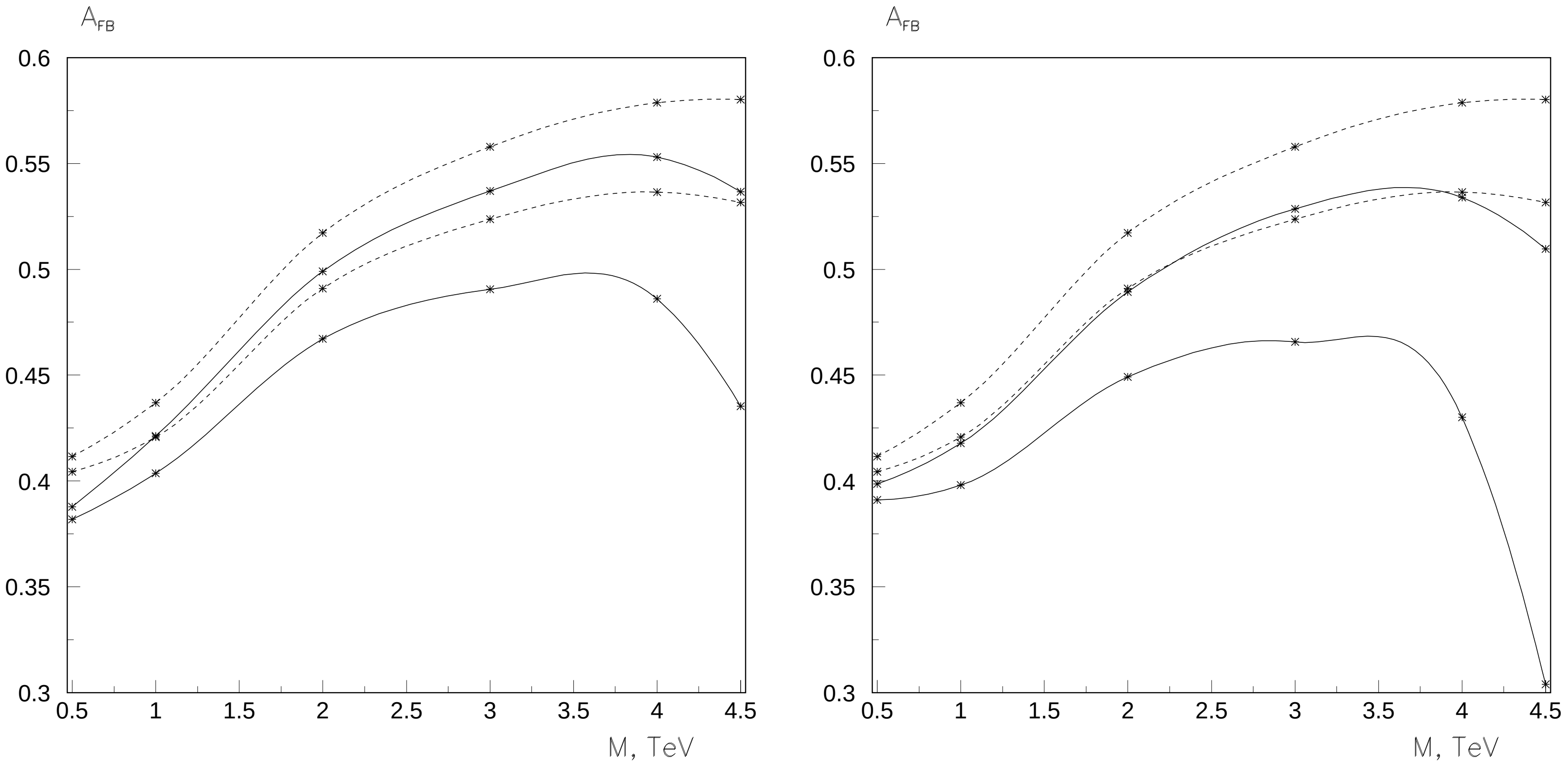


Figure 9: The forward-backward asymmetry $A_{\rm FB}$ in the Born approximation and taking into consideration the rad. corrections as a functions of M. Left picture for $l=\mu$, right one for $l=e$.

Conclusions

- The complete electroweak corrections to the Drell-Yan process at M above 1 TeV have been studied.
- The results for weak part are the compact asymptotic expressions, they expand in the powers (zero, first and second) of electroweak Sudakov logarithms.
- At the parton level we compare the investigated radiative corrections with the existing results and obtain a rather good coincidence at energy higher than 0.5 TeV.
- READY is ready!
- Based on READY Monte Carlo generator READYGEN for simulation of radiative events is the next aim of our working group (in next report of Sasha Ilyichev)

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