Chaotic instantons and tunneling in perturbed double-well system

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Tunneling in double-well potential

- Tunneling phenomena in double-well potential was connected with instantons.
- In perturbed double-well potential a rate of tunneling many orders of magnitude greater than in undriven one (chaos assisted tunneling) [*Lin, Ballentine, 90*].
- The decreasing of tunneling rate was found for specific parameter values of the driving force [*Hänggi et. al., 91*].
- Analytic chaotic instanton approach was proposed in order to describe enhancement of tunneling in one-dimensional spatially periodic potential [*Kuvshinov et. al., 02, 03*].
- Chaotic instanton approach was used for monochromatically driven double-well potential [*Kuvshinov et. al., 06*].

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Our results

- Previous researches investigated oscillating external driving force for double-well potential ($v_{per} \sim cos(\nu t)$).
- We regard perturbation of the kick type ($v_{per} \sim \delta(t - nT)$).
- We use analytic chaotic instanton approach for system under investigation.
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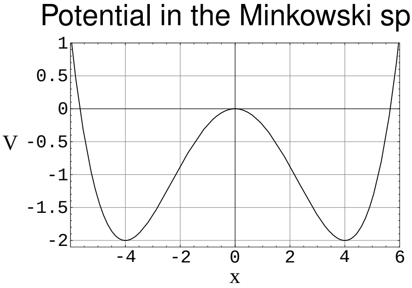
Undriven double-well system - Minkowski space

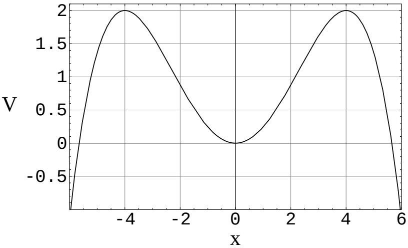
Hamiltonian of the classical particle in the double-well potential in Minkowski space:

$$H_0 = \frac{p^2}{2m} + a_0 x^4 - a_2 x^2,$$

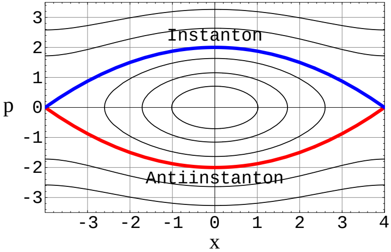
a_0, a_2 - parameters of the potential.



Potential in the Euclidean space



Phase space of undriven system



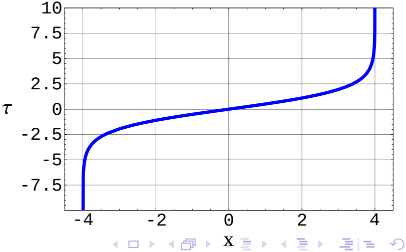
Instanton - $X(\tau) = \sqrt{\frac{a_2}{2a_0}} \tanh\left(\frac{\omega\tau}{2}\right)$

Energy splitting:

$$\Delta E = 2\sqrt{\frac{6}{\pi}}\sqrt{S^{inst}}\, e^{-S^{inst}}$$

$S^{inst} = \frac{2\, a_2^{3/2}\sqrt{m}}{3\, a_0}$ - instanton action

(for details see [*Vainshtein et. al., 82*])



Perturbed double-well system

Hamiltonian of the classical particle in the double-well potential:

$$H_0 = \frac{p^2}{2m} + a_0 x^4 - a_2 x^2,$$

a_0, a_2 - parameters of the potential.
Perturbation has the following form:

$$V_{per} = \varepsilon x \sum_{n=-\infty}^{+\infty} \delta(t - nT),$$

where ε - value of the perturbation, T - period of the perturbation, t - time.
Full Hamiltonian of the system:

$$H = H_0 + V_p.$$

Chaotic instantons in kicked double-well potential

- Perturbation destroy separatrix forming stochastic layer.
- The manifestation of the perturbation is the appearance of a number of the additional solutions of Euclidian equations of motion with energies close to the energy of nonperturbed one-instanton solution and placed inside the stochastic layer.
- In this case

$$x_{chaos} = x_{inst} + \varepsilon \Delta x_{chaos}$$

  solution of the Euclidean equation of motion. Here x_{chaos} and x_{inst} - chaotic (perturbed) and nonperturbed instantons, Δx_{chaos} - stochastic correction.
- Assumptions of chaotic instanton approach [*Kuvshinov et. al., 02, 03*]:
  - uniform stochastic layer,
  - small value $\epsilon < 0.1$ and frequency of perturbation,
  - Euclidean chaotic instanton action is equal to nonperturbed instanton action with some nonmaximal energy.

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Tunneling amplitude

Width of stochastic layer is following

$$\Delta H_s = E_{sep} - E_{bor} \sim \frac{\varepsilon\,\nu^3}{16\sqrt{2}\,\pi^2}\,\frac{m}{\sqrt{a_0 a_2}}\,\frac{1}{sh\left(\frac{\pi}{2}\,\sqrt{\frac{m}{a_2}}\,\nu\right)}.$$

The tunneling amplitude ($\xi = E_{sep} - E$)

$$A = \int_0^{\Delta H_s} d\,\xi \int_{\infty}^{+\infty} d\,c_0 \sqrt{S[x^{inst}(\tau,\xi)]}\,exp(-S[x^{inst}(\tau,\xi)]) \approx$$

$$\approx \sqrt{S^{inst}}\,e^{-S^{inst}}\,\Gamma\,exp\left(\frac{1}{6}(1 + 18 ln\,2)\sqrt{\frac{m}{a_2}}\Delta H_s\right)$$

Ground quasienergy splitting

We could write phenomenological formula for quasienergy splitting

$$\Delta E_Q(\varepsilon,\nu) = 2\sqrt{\frac{6}{\pi}}\sqrt{S^{inst}}\,e^{-S^{inst}}\left(1 + k\frac{\varepsilon\,\nu^3}{sh(\frac{\pi}{2}\sqrt{\frac{m}{a_2}}\nu)}\right),$$

where k - phenomenological parameter. Nonperturbed instanton action

$$S^{inst} = \frac{2\,a_2^{3/2}\sqrt{m}}{3\,a_0}$$

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Main steps of numerical calculations

- Analytical calculation of hamiltonian (H_0) and perturbation (V_{per}) matrix elements using as basis vectors the eigenvectors of a harmonic oscillator.
- Numerical calculation of matrix exponents e^{-iHT} and $e^{-iV_{per}}$.
- Calculation of eigenvalues of the evolution operator $e^{-iHT}e^{-iV}$ which give quasienergy levels.
- The size of the matrices is 100.
- Mathematica 5.2 was used for numerical calculations.

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Quasienergy splitting vs value of perturbation

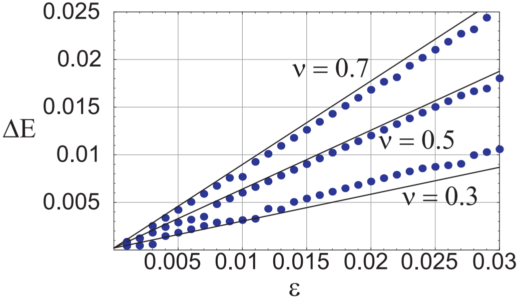
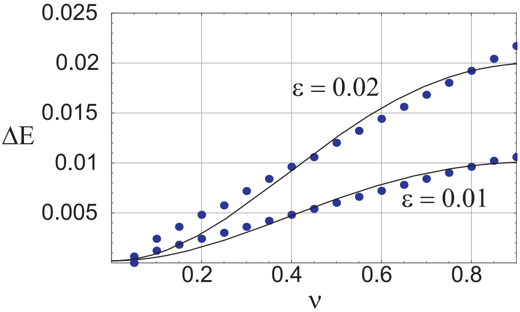


Figure: Quasienergy splitting as a function of the value of perturbation for fixed perturbation frequency. Model parameters: $m = 1, a_0 = \frac{1}{128}, a_2 = \frac{1}{4}$.

Line - phenomenological formula,
points - results of numerical calculations.

Quasienergy splitting vs frequency of perturbation

Figure: Quasienergy splitting as a function of the frequency of perturbation for fixed value of perturbation. Model parameters: $m = 1, a_0 = \frac{1}{128}, a_2 = \frac{1}{4}$.



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- Phenomenological formula for ground quasienergy splitting as a function of value and frequency of perturbation was obtained.
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- There are significant differences between analytical and numerical calculations for low and high perturbation frequencies.

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Thank you for attention