QCD VACUUM,
QUARK CONFINEMENT
AND MESON SPECTRUM

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1. **Particle physics and quantum field theory. QED and QCD.**
2. **Vacuum field - (Anti-)selfdual homogeneous gluon field.**
   Confinement of quarks and gluons
3. **Hadronization in QCD, meson mass equation**
4. **Physical results:**
   (a) **meson masses $J^P = 0^-,\ 1^-,\ 0^+,\ 1^+,\ 2^+,\ 3^-,$**
   (b) **radial excitations of $1^-,\ 2^+$ mesons.**

Quantum field theory - QED and QEWD

$\mathcal{L}$	=	$\mathcal{L}_0$	+	$g\mathcal{L}_I$
		non-interacting particles plane waves		particle interaction $\alpha = \frac{g^2}{4\pi} < 1$

Standard approach in QCD

$\mathcal{L}^{QCD}$	=	$\mathcal{L}_0$	+	$g\mathcal{L}_I$
		non-interacting quarks & gluons plane waves no confinement		quark-gluon interaction $\alpha_s = \frac{g^2}{4\pi} < 1$
Idea	$\longrightarrow$	nonperturbative interaction	$\longrightarrow$	confinement

$\Downarrow$

Phenomenology (symmetries, new parameters)

$\Downarrow$

$\mathcal{L}_{eff}^{hadrons}$	=	$\mathcal{L}_0^{hadrons}$	+	$g_{eff}\mathcal{L}_I^{hadrons}$
effective Lagrangian		non-interacting free hadrons plane waves		hadron interaction

Idea - a vacuum gluon field $B_\mu(x)$ should provide the confinement in QCD

$\mathcal{L}^{QCD}[B]$	=	$\mathcal{L}_0[B]$	+	$g\mathcal{L}_I$
		non-interacting quarks & gluons confinement no plane waves		**quark-gluon interaction** $\alpha_s = \frac{g^2}{4\pi} < 1$

$\Downarrow$

$$\Lambda_{confinement} \sim \Lambda_{hadronization}$$

hadronization = collective-hadron variables
quark-hadron duality
Bethe-Salpeter equation
Parameters: $\{m_q\},\ \alpha_s = \frac{g^2}{4\pi},\ \Lambda_{conf}$

$\Downarrow$

$\mathcal{L}^{hadrons}$	=	$\mathcal{L}_0^{hadrons}$	+	$g_h\mathcal{L}_I^{hadrons}$
		non-interacting free hadrons plane waves		**non-local hadron-hadron interaction** $\alpha_h = \frac{g_h^2}{4\pi} < 1$

QED. Selfdual electromagnetic field with constsnt strength.

$$B_\mu(x) = \Lambda^2\, b_{\mu\nu} x_\nu,$$

$$B_{\mu\nu} = \partial_\mu B_\nu(x) - \partial_\nu B_\mu(x) = -2\Lambda^2\, b_{\mu\nu},$$

$$b_{\mu\nu} = -b_{\nu\mu}, \quad b_{\mu\rho} b_{\rho\nu} = -\delta_{\mu\nu},$$
$$\tilde{b}_{\mu\nu} = \frac{1}{2}\epsilon_{\nu\mu\alpha\beta} b_{\alpha\beta} = \pm b_{\mu\nu}.$$

$$\partial_\mu B_{\mu\nu}(x) = 0$$

QCD. Selfdual gluon field with constsnt strength.

$$\breve{B}_\mu(x) = B^a_\mu(x) t^a, \quad B^a_\mu(x) = \Lambda^2\, n^a\, b_{\mu\nu} x_\nu,$$
$$B^a_{\mu\nu} = \partial_\mu B^a_\nu(x) - \partial_\nu B^a_\mu(x) = -2\Lambda^2\, n^a\, b_{\mu\nu},$$
$$[\breve{B}_\mu(x), \breve{B}_\nu(x)] = 0,$$

$$n^a : \quad (n^a n^a) = 1, \quad (n^a n^b n^c d^{abc})$$

$$\nabla_\mu \breve{B}_{\mu\nu} = \partial_\mu \breve{B}_{\mu\nu} - ig[\breve{B}_\mu(x), \breve{B}_{\mu\nu}(x)] = 0.$$

Fermion propagator in a self-dual field.

$$(\hat{\nabla}(x) - m)S(x - x') = -\delta(x - x'),$$

$$\nabla_\mu(x) = \partial_\mu + i\Lambda^2 \breve{n} b_{\mu\nu} x_\nu.$$

$$S_\pm(x - x') \;=\; e^{i(x_\mu \breve{B}_{\mu\nu} x'_\nu)} \int \frac{dp}{(2\pi)^4} e^{ip(x-x')} \tilde{S}_\pm(p),$$

$$\tilde{S}_\pm(p) = \int\limits_0^1 \frac{du}{2\Lambda^2} e^{-u\frac{p^2}{2\Lambda^2}} \left(\frac{1-u}{1+u}\right)^{\frac{m^2}{4\Lambda^2}}$$

$$\cdot \left\{ i\hat{p} \pm u\breve{n}\gamma_5(\gamma b p) + \frac{m}{1-u^2}\left[1 \mp \gamma_5 u^2 + \frac{i}{2}\breve{n}(\gamma b \gamma) u\right]\right\}.$$

Analytical confinement !

Non-local distributions and Confinement

-

$$\tilde{G}(E) \sim e^{l^2E^2}$$

$$G(t-t') = e^{-\left(l\frac{d}{dt}\right)^2} \cdot \delta(t-t') = \int_{-\infty}^{\infty} \frac{dE}{2\pi} e^{-iE(t-t')+l^2E^2}.$$

$$J_{out}(t) = \int_{-\infty}^{\infty} G(t-t')J_{in}(t') = \int_{-\infty}^{\infty} \frac{dE}{2\pi} e^{-iEt} \cdot e^{l^2E^2} \tilde{J}_{in}(E).$$

$$|\tilde{J}(E)| \leq Ce^{-|E|^\sigma}, \quad \sigma > 2.$$

$$J_{in}(t) \sim Ce^{-b|t|^\rho} \implies J_{out}(t) \sim Ce^{-b|t|^\rho}, \quad 1 < \rho < 2$$

$$\text{Confinement} \Rightarrow \begin{cases} 1. \ \text{Two static quarks} + \text{Potential } V(r) \sim r \\ \\ 2. \ \text{Quarks are fluctuations in space and time} \end{cases}$$

Vacuum energy in a self-dual homogeneous field $B_\mu(x)$

$$\begin{aligned}
E_{fermion} &\sim \frac{4\Lambda^4}{12\pi^2}\ln\left[1+\left(\frac{\Lambda^2}{M^2}\right)^2\right],\\
E_{boson} &\sim -\frac{2\Lambda^4}{12\pi^2}\ln\left[1+\left(\frac{\Lambda^2}{m^2}\right)^2\right],\\
E_{gluon} &\sim \frac{33\Lambda^4}{12\pi^2}\ln\left(\frac{\Lambda^2}{\Lambda^2_{QCD}}\right),
\end{aligned}$$

$$\begin{aligned}
E_{QED} &= \sum_F E_F(\Lambda)+\sum_B E_B(\Lambda)\\
&\sim \frac{\Lambda^4}{12\pi^2}\left[4\sum_F \ln\left[1+\left(\frac{\Lambda^2}{M_F^2}\right)^2\right]-2\sum_B \ln\left[1+\left(\frac{\Lambda^2}{m_B^2}\right)^2\right]\right]
\end{aligned}$$

$$\begin{aligned}
E_{QCD} &= E_g(\Lambda)+\sum_q E_q(\Lambda)\\
&\sim \frac{\Lambda^4}{12\pi^2}\left[33\ln\left(\frac{\Lambda^2}{\Lambda^2_{QCD}}\right)+4\sum_q \ln\left[1+\left(\frac{\Lambda^2}{M_q^2}\right)^2\right]\right]
\end{aligned}$$

Stability

$$\text{QED}:\ \ 2N_F > N_B,\qquad \sum_F\frac{2}{M_F^2} > \sum_B\frac{1}{m_B^2},\ \Rightarrow\ \Lambda=0.$$

$$\text{QCD}:\ \ \Lambda>0,\qquad Always!.$$

$$\mathrm{QCD}, \quad \mathcal{L} = -\frac{1}{8}\mathrm{Tr}\,\breve{G}^2_{\mu\nu} + \bar{q}(i\hat{\nabla} - m)q$$
$$\mathrm{QCD\ vacuum} \ \rightarrow \ \breve{B}_\mu(x)$$

$$\downarrow$$

$$\frac{g^2}{2}(\bar{q}\gamma_\mu t^a q)_{x_1} D^{ab}_{\mu\nu}(x_1, x_2)(\bar{q}\gamma_\nu t^b q)_{x_2}$$

$$\downarrow$$

Fierz transformation

$$\frac{2g^2}{9}\sum_J \mathcal{J}_J(x_1, x_2)\mathcal{J}_J(x_2, x_1) = \frac{g^2}{2}\sum_J \mathcal{J}_J(x, y)\mathcal{J}_J(x, y)$$

$$\downarrow$$

Quantum numbers and vertex functions

$$\mathcal{J}_J(x, y) = \sqrt{D(y)}(\bar{q}(x + \frac{y}{2})\Gamma_J q(x - \frac{y}{2})) = \sum_Q J_{JQ}(x)U_Q(y)$$
$$J_{JQ}(x) = \bar{q}(x)V_{JQ}(\overleftrightarrow{\partial})q(x), \quad V_{JQ}(\overleftrightarrow{\partial}) = \int dy\, U_{JQ}(y)\sqrt{D(y)}e^{\frac{y}{2}\overleftrightarrow{\partial}}$$

$$\downarrow$$

$$L_I = \frac{g^2}{2}\sum_{JQ}\int dx\, J^2_{JQ}(x)$$

$$\downarrow$$

$$e^{\frac{g^2}{2}(J^2_{JQ})} = \int DB_Q\, e^{-\frac{1}{2}(B^2_{JQ}) + g(B_{JQ}J_{JQ})} \ \rightarrow \ \text{integration over } \bar{q} \text{ and } q$$

$$\downarrow$$

$$\int DB_{JQ}\, e^{-\frac{1}{2}(B^2_{JQ}) + \mathrm{Tr}\ln[1 + g(B_{JQ}V_{JQ})S]}$$
$$\rightarrow \int DB_{JQ}\, e^{-\frac{1}{2}(B_{JQ}[I - g^2\mathrm{Tr}(V_{JQ}SV_{JQ}S)]B_{JQ}) + W_I[B]}$$

$$\downarrow$$

Bethe − Salpeter equation

$$g^2\mathrm{Tr}(V_{JQ}SV_{J'Q'}S) = (U_{JQ}\Pi U_{J'Q'}) = \lambda_{JQ}(-p^2)\delta_{JJ'}\delta_{QQ'}$$

$$\downarrow$$

$$1 = \lambda_{JQ}(M^2_{JQ}),$$

$$\downarrow$$

$$W_I[B] = \frac{g^3}{3}\mathrm{Tr}[(B_{JQ}V_{JQ})S(B_{JQ}V_{JQ})S(B_{JQ}V_{JQ})S] + O(B^4)$$

Meson masses

$$I. \quad \Lambda_{hadronization} \approx \Lambda_{confinement}$$

$$II. \quad Analytical\ confinement$$

$$III. \quad Bethe - Salpeter\ \ equation$$

$$IV. \quad \mathcal{M}_Q(x) = \left(\bar{q}_1(x) V_Q \left(\overleftrightarrow{\partial}_x\right) q_2(x)\right)$$

$$M_Q(m_1, m_2) = \mathcal{M}(m_1, m_2, A_Q)$$

$$= \ (m_1 + m_2) \left[1 + \frac{A_Q}{(m_1^2 + 1.13 m_1 m_2 + m_2^2)^{0.625}}\right],$$

$$Q = (J^P, n)$$

$$n = 0 \quad J^P = 0^-,\ 1^-,\ 0^+,\ 1^+,\ 2^+,\ 3^-$$

$$n > 0 \quad J^P = 1^-,\ 2^+$$

$$(u = d,\ s,\ ,c\ ,b)$$

$$1 = \frac{\alpha_s}{\pi} \iint dy_1 dy_2 V_Q(y_1) \Pi_Q(y_1 - y_2|p) V_Q(y_2).$$

$$1 \ = \ \frac{\alpha_s}{\pi} \iint\limits_0^1 du_1 du_2 P_Q(...) e^{\frac{(m_1+m_2)^2}{2\Lambda^2} E(\mu,\mu_1,\mu_2,u_1,u_2)}.$$

$$E(\mu, \mu_1, \mu_2, u_1, u_2) = \mu^2 \cdot \frac{u_1 u_2 + 2(\mu_1^2 u_1 + \mu_2^2 u_2)}{u_1 + u_2 + 2}$$
$$+ \frac{\mu_1^2}{2} \ln\left(\frac{1 - u_1}{1 + u_1}\right) + \frac{\mu_2^2}{2} \ln\left(\frac{1 - u_2}{1 + u_2}\right).$$

$$\mu = \frac{M}{m_1 + m_2}, \quad \mu_j = \frac{m_j}{m_1 + m_2}, \ \ j = 1, 2$$
$$1 \le \mu = \frac{M}{m_1 + m_2} \le 1 \div 2.5; \quad \mu_1 + \mu_2 = 1.$$

$$\mu = \frac{M}{m_1 + m_2} < 1, \quad M < m_1 + m_2 \quad \Longrightarrow \ \pi \ \ K$$

$$M > m_1 + m_2, \quad e^{\frac{(m_1+m_2)^2}{2\Lambda^2}E(\mu,\mu_1,\mu_2)} > 1$$

$$\iint_0^1 du_1 du_2 P_Q(...) e^{\frac{(m_1+m_2)^2}{2\Lambda^2}E(...)} \approx C_Q e^{\frac{(m_1+m_2)^2}{2\Lambda^2}\mathcal{E}(M,m_1,m_2)}$$

$$\mathcal{E}(M, m_1, m_2) = \max_{u_1,\, u_2} E(\mu, \mu_1, \mu_2, u_1, u_2)$$

$$= \max_{u_1,u_2} \left[\mu^2 \cdot \frac{u_1 u_2 + 2(\mu_1^2 u_1 + \mu_2^2 u_2)}{u_1 + u_2 + 2} \right.$$
$$\left. + \frac{\mu_1^2}{2} \ln\left(\frac{1-u_1}{1+u_1}\right) + \frac{\mu_2^2}{2} \ln\left(\frac{1-u_2}{1+u_2}\right) \right].$$

$$1 \sim \frac{\alpha_s}{\pi} C_Q e^{\frac{(m_1+m_2)^2}{2\Lambda^2}\mathcal{E}(M,m_1,m_2)}.$$

$$M_Q(m_1, m_2) = M(m_1, m_2, A_Q)$$

$$\approx (m_1 + m_2) \left[1 + \frac{A_Q}{(m_1^2 + 1.13 m_1 m_2 + m_2^2)^{0.625}} \right].$$

Meson masses.

$$Q = J^P \;\Rightarrow\; \begin{pmatrix} uV_Q\bar{u} & uV_Q\bar{s} & uV_Q\bar{c} & uV_Q\bar{b} \\ & sV_Q\bar{s} & sV_Q\bar{c} & sV_Q\bar{b} \\ & & cV_Q\bar{c} & cV_Q\bar{b} \\ & & & bV_Q\bar{b} \end{pmatrix}.$$

$$P(0^-) = \begin{pmatrix} \eta(547) & K(494) & D(1869) & B(5279) \\ & \eta'(957) & D_s(1968) & B_s(5369) \\ & & \eta_c(2979) & B_c(6400) \\ & & & \eta_b(9300) \end{pmatrix};$$

$$V(1^-) = \begin{pmatrix} \omega(782) & K^*(892) & D^*(2007) & B^*(5325) \\ & \phi(1020) & D_s^*(2112) & - \\ & & J/\psi(3100) & - \\ & & & \Upsilon(9460) \end{pmatrix};$$

$$S(0^+) = \begin{pmatrix} f_0(980) & - & - & - \\ & f_0(1370) & - & - \\ & & \chi_{c0}(3415) & - \\ & & & \chi_{b0}(9893) \end{pmatrix};$$

$$A(1^+) = \begin{pmatrix} a_1(1260) & K_1(1330) & - & - \\ & f_1(1420) & - & - \\ & & \chi_{c1}(3510) & - \\ & & & \chi_{b1}(9892) \end{pmatrix};$$

$$D(2^+) = \begin{pmatrix} f_2(1270) & K_2^*(1430) & D_2^*(2460) & - \\ & f_2'(1525) & - & - \\ & & \chi_{c2}(3556) & - \\ & & & \chi_{b2}(9912) \end{pmatrix}.$$

$$T(3^-) = \begin{pmatrix} \omega_3(1670) & K_3^*(1780) \\ & \phi_3(1850) \end{pmatrix}.$$

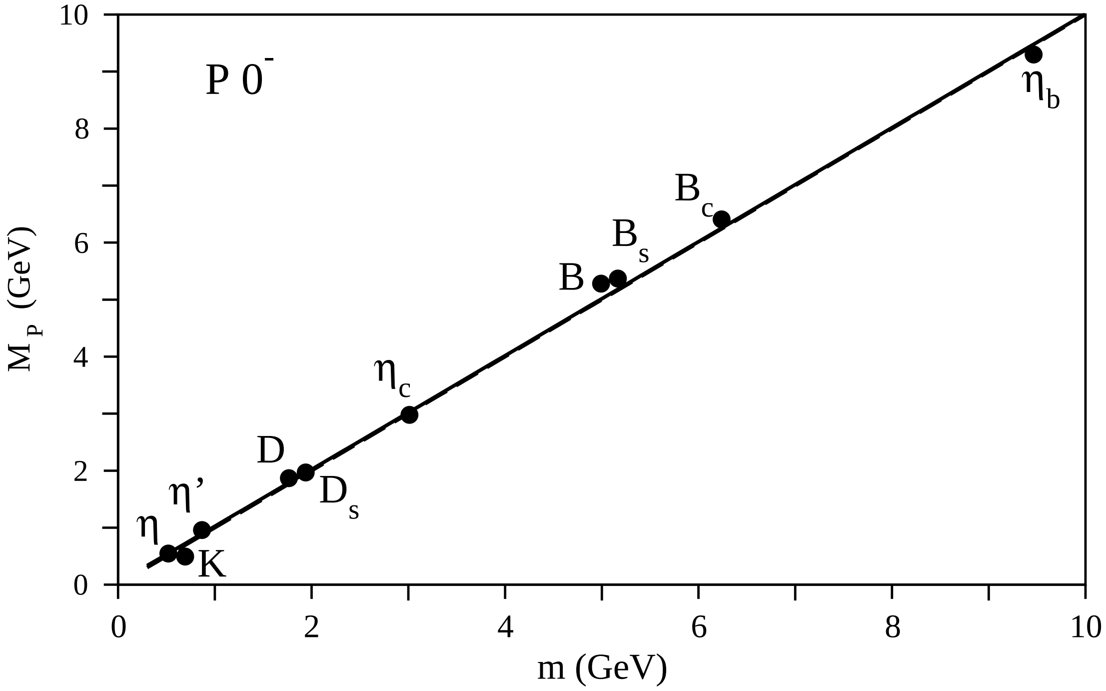
$$m_u = m_d,\ m_s,\ m_c,\ m_b \quad - \ 4 \ - \ \text{parameters}$$

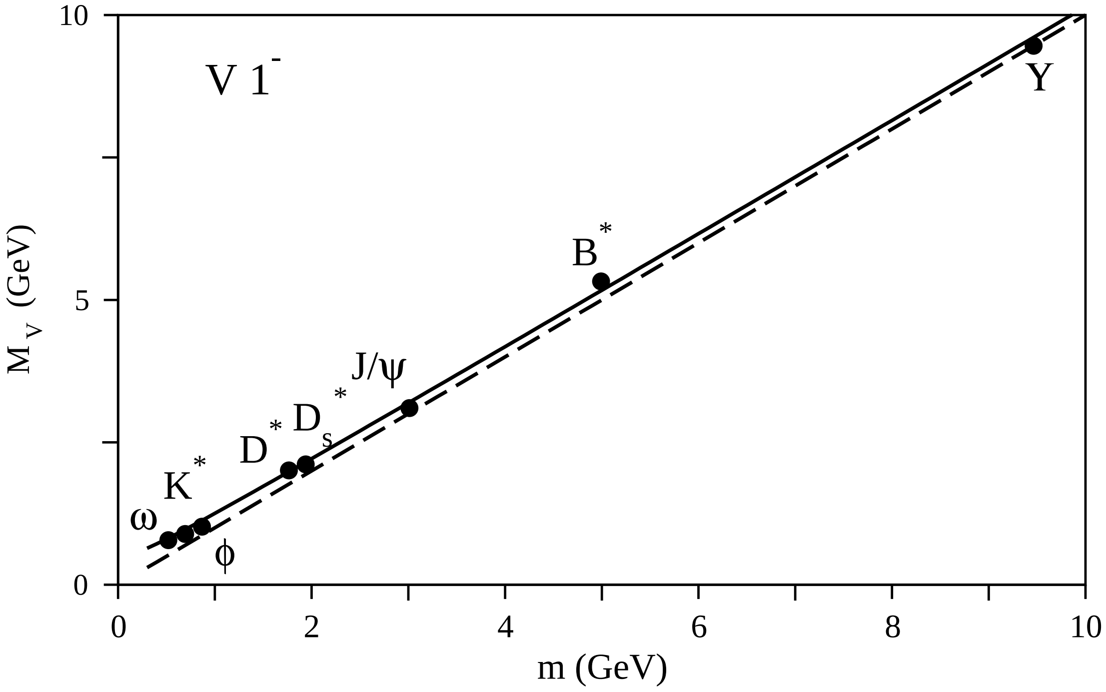
$$A_P,\ A_V,\ A_S,\ A_A,\ A_D,\ A_T \quad - \ 6 \ - \ \text{parameters}$$

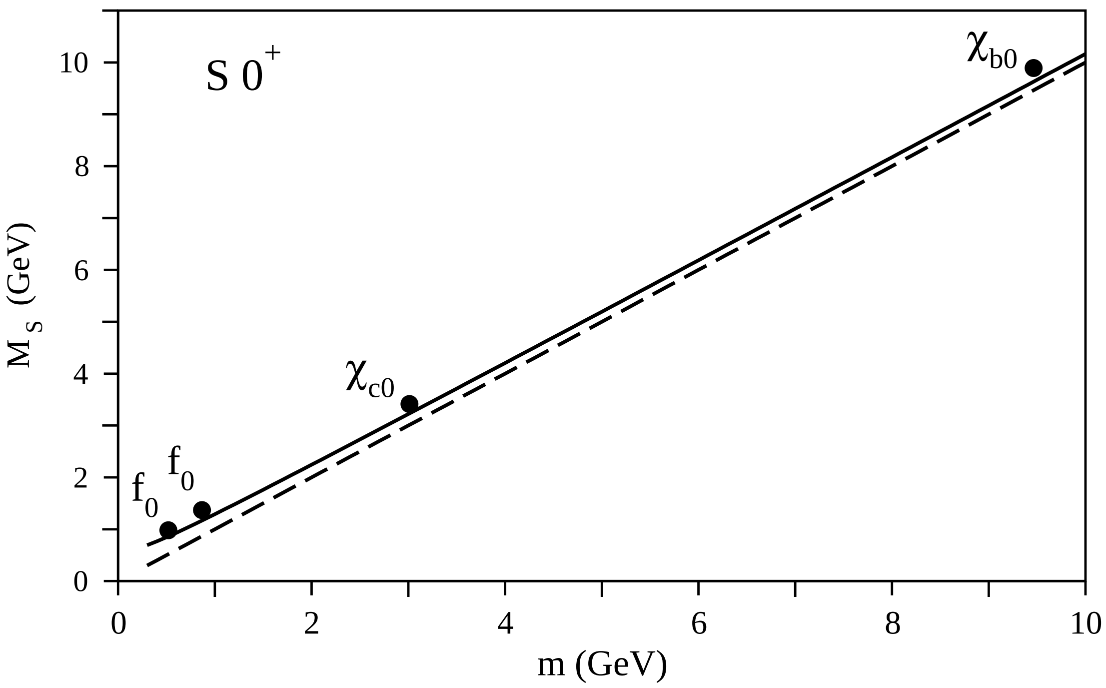
$$O_Q = \frac{1}{N_Q} \sum_{m_1, m_2, A_Q} \left[1 - \frac{M_{Q,m_1,m_2}(m_1, m_2, A_Q)}{(M_{Q,m_1,m_2})_{\text{exp}}} \right]^2$$

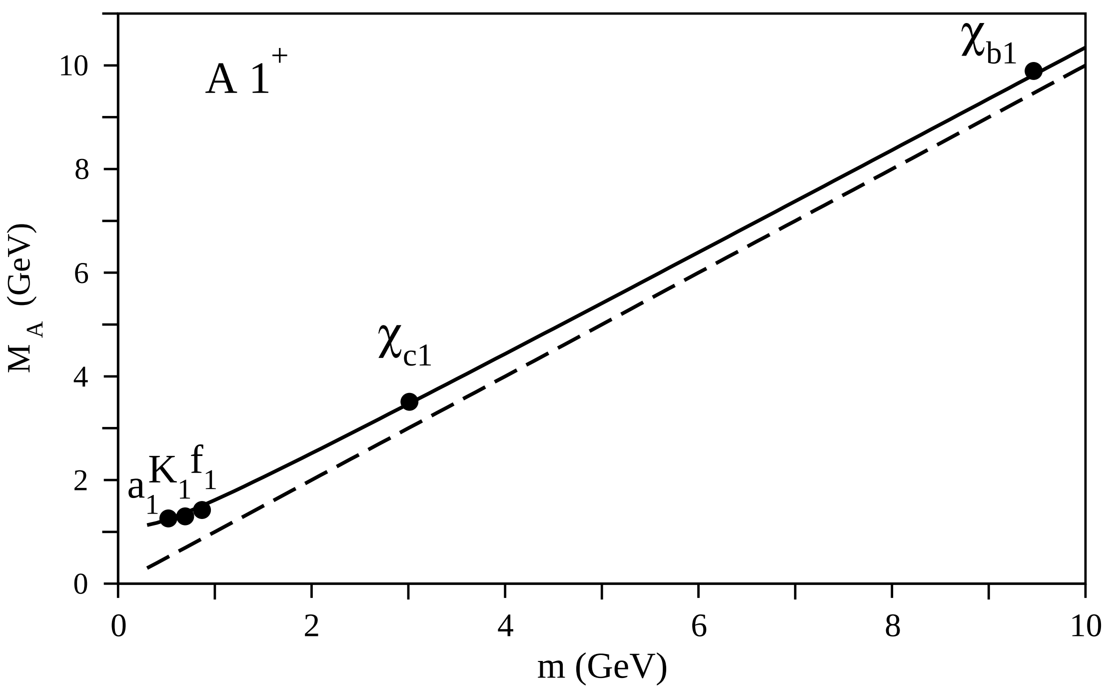
m_f	m_u	m_s	m_c	m_b
Mev	**260**	**434**	**1506**	**4732**

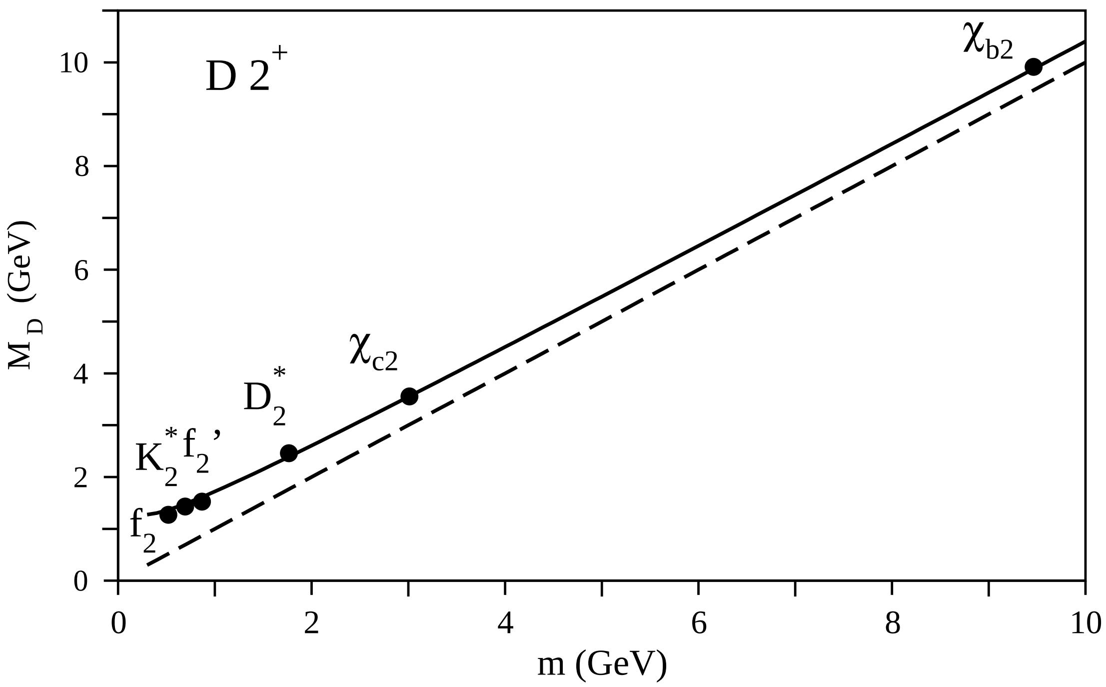
Q	$P = 0^-$	$V = 1^-$	$S = 0^+$	$A = 1^+$	$D = 2^+$	$T = 3^-$
A_Q	**0.0216**	**0.217**	**0.249**	**0.527**	**0.618**	**0.838**
O_Q	**0.011**	**0.0026**	**0.012**	**0.0023**	**0.0019**	**0.00016**

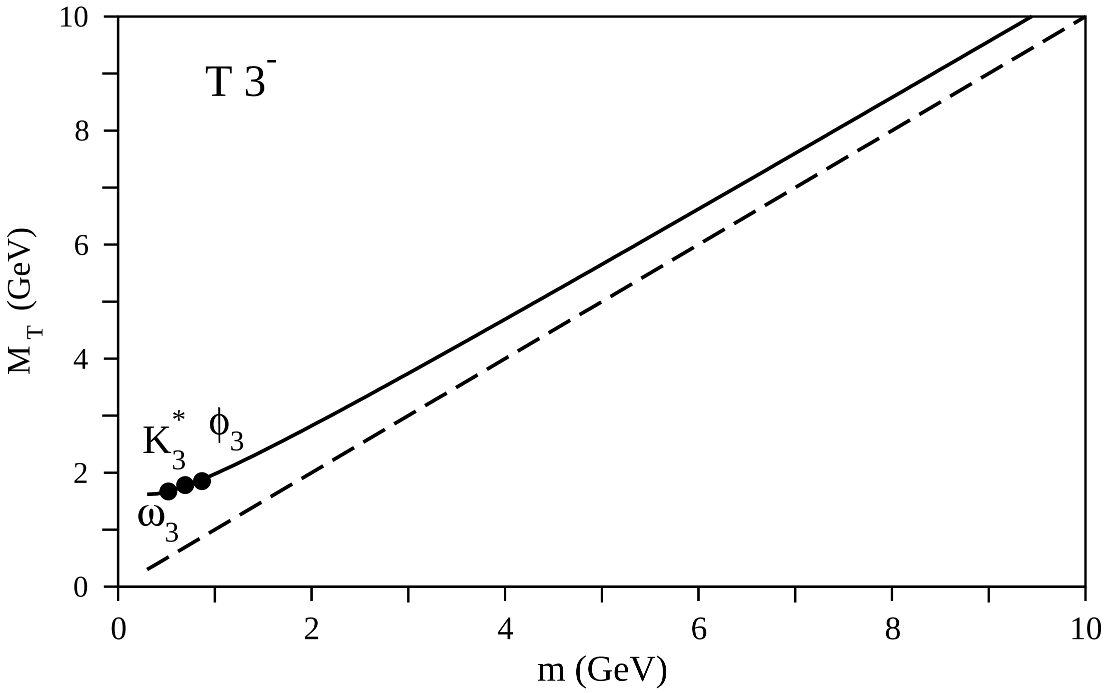

P 0-
M_P (GeV)
m (GeV)
η
η'
K
D
D_s
$η_c$
B
B_s
B_c
$η_b$
0
2
4
6
8
10


V 1-
M_V (GeV)
m (GeV)
ω
K^*
φ
D^*
D_s^*
J/ψ
B^*
Y
0
5
10
2
4
6
8


S 0^+
χ_{b0}
χ_{c0}
f_0
f_0
M_S (GeV)
m (GeV)
0
2
4
6
8
10


A 1^+
χ_{b1}
χ_{c1}
a_1
K_1
f_1
M_A (GeV)
m (GeV)
0
2
4
6
8
10


D 2+
M_D (GeV)
m (GeV)
χb2
χc2
D2*
K2* f2'
f2
0
2
4
6
8
10


T 3-
M_T (GeV)
m (GeV)
K3* φ3
ω3
0
2
4
6
8
10

Radial excitations for vector $\mathbf{1^-}$ ***mesons.***

n	$\rho = u\bar{u}$	$\psi = c\bar{c}$	$\Upsilon = b\bar{b}$	A_V	O_V
0	$\rho(770)$ 778	$J/\psi(3100)$ 3178	$\Upsilon(1S)(9460)$ 9589	0.188	0.0079
1	$\rho(1450)$ 1323	$\psi(2S)(3655)$ 3530	$\Upsilon(2S)(10023)$ 9853	0.585	0.0037
2	$\rho(1700)$ 1699	$\psi(3770)$ 3772	$\Upsilon(3S)(10365)$ 10035	0.859	0.0011
3	$\rho(1900)$ 2081	$\psi(4040)$ 4018	$\Upsilon(4S)(10580)$ 10220	1.137	0.0034
4	$\rho(2150)$ 2390	$\psi(4160)$ 4217	$\Upsilon(10860)$ 10370	1.362	0.0049
5	$\rho(-)$ 2698	$\psi(4415)$ 4416	$\Upsilon(11020)$ 10518	1.574	—

$$\boldsymbol{Tensor\ mesons\ 2^+}$$

$$D1(2^+,1) = \begin{pmatrix} f_2(1565) & K_2^*(1980) & - & - \\ & f_2(2010) & - & - \\ & & - & - \\ & & & \chi_{b2}(2P)(10268) \end{pmatrix}.$$

	$n=0$	$n=1$
$u\bar{u}$	$f_2(1270)$ - 1336	$f_2(1565)$ - 1737
$u\bar{s}$	$K_2^*(1430)$ - 1445	$K_2^*(1980)$ - 1818
$s\bar{s}$	$f_2'(1525)$ - 1586	$f_2(2010)$ - 1939
$s\bar{c}$	$D_2^*(2460)$ - 2320	$-$ - 2772
$c\bar{c}$	$\chi_{c2}(1P)(3556)$ - 3537	$-$ - 3796
$b\bar{b}$	$\chi_{b2}(1P)(9912)$ - 9859	$\chi_{b2}(2P)(10268)$ - 10053
A_D	0.594	0.886
O_D	0.0013	0.0051

Light mesons

$$\pi(138) = (u\bar{u}) \qquad K(494) = (u\bar{s})$$

$$M_\pi = 138 < 2m_u = 520, \qquad M_K = 494 < m_u + m_s = 694.$$

$$\left(\frac{F(M_\pi, m_u, m_u)}{F(M_K, m_u, m_s)}\right)^2 = 1.26, \quad (\sim 30\%).$$

Conclusion

- **Confinement - quarks and gluons are fluctuations in time-space.**
- **Stability in a self-dual homogeneous field:**
  - **no confinement in QED - all basic particles are fermions and electron has the lowest mass;**
  - **confinement in QCD - gluons have zero mass.**
- **The structure of the quark propagetor in gluon vacuum field predetermines main features of meson spectrum.**
- **The formula**

$$M_Q(m_1, m_2) \approx (m_1 + m_2)\left[1 + \frac{A_Q}{(m_1^2 + 1.13 m_1 m_2 + m_2^2)^{0.625}}\right].$$

  correctly describes the mass dependence of mesons on masses of constituent quarks.
- **The (anti)self-dual homogeneous field is a good candidate to be the vacuum gluon field.**