Acceleration of the Universe expansion from the point of view of quantum field theory in curved space.

S.L. Cherkas
Institute for Nuclear Problems, Minsk, Belarus

V.L. Kalashnikov
Technische Universität Wien, Vienna

Old-New problem

Cosmological constant Λ

and

Observed acceleration of the Universe expansion

Perlmutter S.J. et al, Astrophys. J. **517** 565 (1999)

Riess A. et al, Astron. J. **116** 1009 (1998)

$$\textit{energy density of vacuum}: \qquad \rho_{vac} = \frac{\Lambda}{8\pi G} \qquad ?????????????$$

G is Newton gravitational constant.

Assumptions

Quasiclassical gravity is valid

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G < T_{\mu\nu} >$$

That is expectation value of $T_{\mu\nu}$ over vacuum state is used in the Einstein equations.

☐ Naive consideration, using ultra violet momentum cut-off in Friedman equation gives $\rho_{vac} \sim k_{max}^4 \sim M_p^4$, where M_p is the Plank mass. This is too large!

☐ Using dimensional regularization of the diverging quantities one has $\rho_{vac} \sim L^{-4}$, where L is the "size" of the Universe an order of inverse curvature radius, or its inverse Hubble constant $L \sim H^{-1}$. In this case it is too tiny!

☐ Experiment is consistent with the estimate $\rho_{vac} \sim M_p^2 L^{-2} \sim M_p^2 H^2$.

Motivation

It is evident, that some renormalization is needed in Friedman equation. to escape cosmological constant problem.

From the other hand after advance in the string theory, it is possible to guess that the UV cutoff at the Planck level may figurate in the expression for the vacuum energy. Thus, renormalizations involving the UV cutoff and providing a reasonable value of the cosmological constant are acceptable in principle.

Thus we'll trying to build a theory of a cosmological constant including ultraviolet (UV) cut off of momentums.

Basic equations

Let us write down the system of equations (flat Universe)

$$-\frac{1}{2}M_p^2 a'^2 + \rho a^4 = const,$$
$$M_p^2 a'' = (\rho - 3p)a^3 \qquad (1)$$

for the scale factor $a(\eta)$, corresponding the metric $ds^2 = a^2(\eta)(d\eta^2 + d\mathbf{r}^2)$, where η is the conformal time connected with the proper time t as $dt = a(\eta)d\eta$ corresponding the metric $ds^2 = dt^2 + a^2(t)d\mathbf{r}^2$.
the density of a matter ρ and the pressure p of the massless scalar field:

$$\rho_\phi = \frac{1}{V}\int_V \left(\frac{\phi'^2}{2a^2} + \frac{(\boldsymbol{\nabla}\phi)^2}{2a^2}\right) d^3\mathbf{r},$$
$$p_\phi = \frac{1}{V}\int_V \left(\frac{\phi'^2}{2a^2} - \frac{(\boldsymbol{\nabla}\phi)^2}{6a^2}\right) d^3\mathbf{r}. \qquad (2)$$

Quantum scalar field satisfies to the equation

$$\hat{\phi}'' + 2\frac{a'}{a}\hat{\phi}' - \Delta\hat{\phi} = 0. \tag{3}$$

Scalar field can be decomposed in a complete set of the modes $\phi(\mathbf{r}) = \sum_{\mathbf{k}} \phi_{\mathbf{k}} e^{i\mathbf{kr}}$ and quantization of the modes consists in postulating (see Birrel and Davis book)

$$\hat{\phi}_{\mathbf{k}} = \hat{\mathrm{a}}^{+}_{-\mathbf{k}}\chi^{*}_{k}(\eta) + \hat{\mathrm{a}}_{\mathbf{k}}\chi_{k}(\eta), \tag{4}$$

where complex functions $\chi_k(\eta)$ satisfy the relations:

$$\begin{gathered} \chi''_k + k^2\chi_k + 2\frac{a'}{a}\chi'_k = 0, \\ a^2(\eta)(\chi_k\,{\chi'_k}^{*} - \chi^{*}_k\,\chi'_k) = i. \end{gathered} \tag{5}$$

The adiabatic approximation

$$\chi_k(\eta) = \frac{\exp\left(-i\int_0^\eta \sqrt{k^2 - \frac{a''(\tau)}{a(\tau)}}\, d\tau\right)}{\sqrt{2}a(\eta)\sqrt[4]{k^2 - \frac{a''(\eta)}{a(\eta)}}} \tag{6}$$

allows calculating the difference of the kinetic and potential energies of field oscillators up to the second-order terms:

$$(\rho_\phi - 3p_\phi)a^3 = \int \left(a < 0|\hat{\phi}'^2|0 > -a < 0|(\nabla\hat{\phi})^2|0 >\right) d^3\mathbf{r} =$$

$$\sum_{\mathbf{k}} a < 0|\hat{\phi}'_{\mathbf{k}}\hat{\phi}'_{-\mathbf{k}}|0 > -k^2 a < 0|\hat{\phi}_{\mathbf{k}}\hat{\phi}_{-\mathbf{k}}|0 >= \sum_{\mathbf{k}} a({\chi'_k}^*\chi'_k - k^2\chi_k^*\chi_k) \approx$$

$$\frac{1}{2}\left(-\frac{a''}{a^2} + \frac{{a'}^2}{a^3}\right)\sum_{\mathbf{k}}\frac{1}{k} + O({a'}^3) + O(a'a'') + O(a'''),$$

where we imply that a' is the first-order quantity, a'' is the second-order one, a''' is the third-order one and so on.

Self consistent equation for the Universe scale factor

$$M_p^2 a'' = \frac{1}{2}\Omega_m M_p^2 \mathcal{H}^2 a_0 + \frac{1}{2}\left(\frac{a''}{a^2} - \frac{a'^2}{a^3}\right)\sum_{\mathbf{k}} \frac{1}{k}. \qquad (8)$$

Eq. (8) can be integrated up to the equation.

$$a'^2 = a_0^2 \mathcal{H}^2 \frac{S_0 - 1 - \Omega_m (a/a_0 - 1)}{S_0 a_0^2/a^2 - 1}, \qquad (9)$$

where the parameter S_0, from the one hand, is determined by the UV cut-off κ_{max} of the physical momentums $\kappa = k/a_0$

$$S_0 = \frac{1}{2M_p^2 a_0^2}\sum_{\mathbf{k}} \frac{1}{k} = \frac{1}{M_p^2 a_0^2 (2\pi)^3}\int \frac{d^3\mathbf{k}}{2k} = \frac{\kappa_{max}^2}{8\pi^2 M_p^2}$$

and, from the other hand, is connected with the present day deceleration parameter q_0 as $S_0 = \frac{2q_0 - 2 + \Omega_m}{2q_0}$.

Investigation of the equation

Eq. (9) can also be rewritten in a cosmic time $dt = a\,d\eta$

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = H_0^2 \frac{(S_0 + \Omega_m - 1)a_0^4 a^{-4} - \Omega_m a_0^3 a^{-3}}{S_0 a_0^2 a^{-2} - 1}, \qquad (10)$$

which gives $a(t) \approx a_0 H_0 \sqrt{\frac{S_0+\Omega_m-1}{S_0}}\, t$ in the vicinity of $t = 0$ (i.e. in the conformal time $a(\eta) = \mathcal{H}\sqrt{\frac{S_0+\Omega_m-1}{S_0}} \exp\left(\mathcal{H}\sqrt{\frac{S_0+\Omega_m-1}{S_0}}\,\eta\right)$). During the further evolution deceleration parameter

$$q(z) = -\frac{\ddot{a}a}{\dot{a}^2} = \frac{1+z}{H}\frac{dH(z)}{dz} - 1 = \frac{2 - 2z(\Omega_m - 1) - \Omega_m + S_0(z+1)(z\Omega_m + \Omega_m - 2)}{2\left(S_0(z+1)^2 - 1\right)\left(zS_0 + S_0 - z + z\Omega_m - 1\right)}, \qquad (11)$$

comes from zero to the present day negative value at small red shifts $z = a_0/a - 1$.

Our model — Vacuum Fluctuation Domination Model

□ Vacuum dominates at all the stages of the Universe expansion. That is radiation domination epoch $a(t) \sim t^{1/2}$, and matter domination epoch $a(t) \sim t^{2/3}$ are absent!

□ Instead $a(t) \sim t$ near $t = 0$ and then Universe began to expand faster.

S.L. Cherkas and V.L. Kalashnikov, gr-qc/0604020, gr-qc/0610148, *JCAP* **0701**, 028 (2007)

Observational Constraints on the Nature of the Dark Energy: Supernova Survey

Luminosity distance

$$d_L(z) = (1+z)\int_0^z \frac{dz'}{H(z')}. \qquad (12)$$

Observed quantity

$$\mu(z) = 5\log\left(\frac{d_L(z)}{10\ \mathrm{pc}}\right). \qquad (13)$$

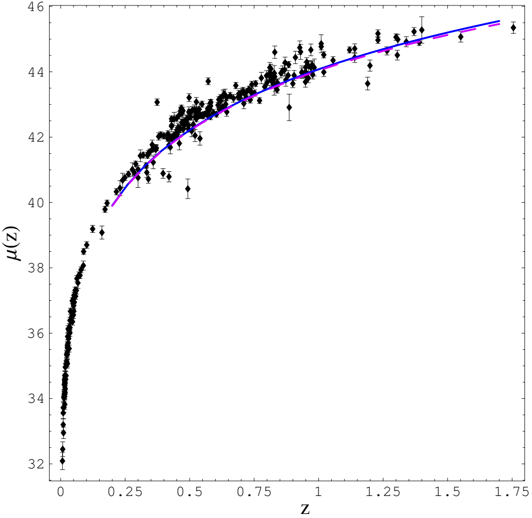


Figure: Riess et al. (2007), Astrophysical Journal vol. 656

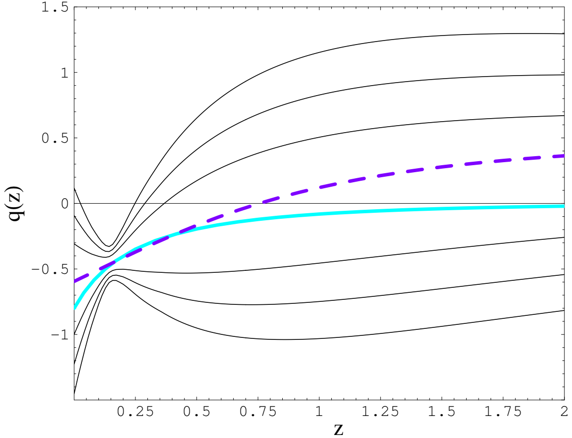
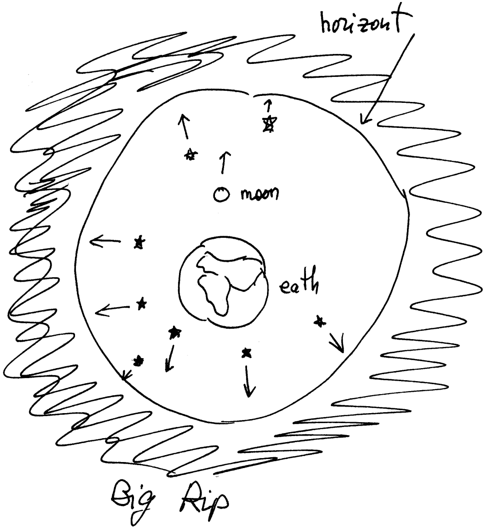


Figure: Vacuum Domination Model curve (blue) for the acceleration parameter evolution and that of ΛCDM (dashed) put on the 1σ, 2σ, 3σ error channels (thin lines) of the reconstruction Y. Gong and A. Wang, *Phys.Rev. D* **73**, 083506 (2006) from the 115 SN Ia data with the parametrization $q(z) = \frac{1}{2} + \frac{q_1 z + q_2}{(1+z)^2}$.

Big Rip



To summarize

☐ It is considered Vacuum Domination Model S.L. Cherkas and V.L. Kalashnikov, gr-qc/0604020, gr-qc/0610148, *JCAP* **0701**, 028 (2007). In this model the Universe acceleration results from the vacuum fluctuations of the fundamental scalar fields.

☐ The calculation shows that the VFDM curve depends on the amount of non-relativistic matter (dust) Ω_m weaker than it takes a place in the ΛCDM case.

☐ Another difference between the models is that VFDM does not predict the change from a deceleration to an acceleration in the past.