Neutrino in the field of the plane electromagnetic wave

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The object of our investigation is the system

$$\nu_l = \begin{pmatrix} \nu_{eL} \\ \nu_{eR} \\ \nu_{\mu L} \\ \nu_{\mu R} \end{pmatrix}. \qquad (1)$$

The wave functions of the flavor basis ν_l and of the mass one ν_i are connected by the relation

$$\nu_i = \mathcal{U}\nu_l = \begin{pmatrix} c_\theta & 0 & s_\theta & 0 \\ 0 & c_\theta & 0 & s_\theta \\ -s_\theta & 0 & c_\theta & 0 \\ 0 & -s_\theta & 0c_\theta & \end{pmatrix} \nu_l, \qquad (2)$$

where $s_\theta = \sin\theta, c_\theta = \cos\theta$, and θ is the neutrino mixing angle in vacuum.

In the free case for the mass eigenstates the evolution of the neutrino flux in ultrarelativistic limit ($E_i \gg m_i$) is given through the Schrodinger-like equation

$$i\frac{d}{dz}\nu_i(z) = \mathcal{H}\nu_i(z), \qquad (3)$$

where

$$\mathcal{H} = \begin{pmatrix} E_1 & 0 & 0 & 0 \\ 0 & E_1 & 0 & 0 \\ 0 & 0 & E_2 & 0 \\ 0 & 0 & 0 & E_2 \end{pmatrix} \simeq |\mathbf{p}| + \frac{1}{2|\mathbf{p}|}\begin{pmatrix} m_1^2 & 0 & 0 & 0 \\ 0 & m_1^2 & 0 & 0 \\ 0 & 0 & m_2^2 & 0 \\ 0 & 0 & 0 & m_2^2 \end{pmatrix}.$$

Taking into account interaction of the neutrino flux with a matter and passing to the weak eigenstates, we obtain the evolution equation in the form

$$i\frac{d}{dz}\nu_l(z) = \mathcal{H}_{int}^{mat}\nu_l(z) =$$

$$= \frac{1}{4E}\begin{pmatrix} -\Delta m_{12}^2 c_{2\theta} + 4E(L_e + L_n) & 0 & \Delta m_{12}^2 s_{2\theta} & 0 \\ 0 & -\Delta m_{12}^2 c_{2\theta} & 0 & \Delta m_{12}^2 s_{2\theta} \\ \Delta m_{12}^2 s_{2\theta} & 0 & \Delta m_{12}^2 c_{2\theta} + 4EL_n & 0 \\ 0 & \Delta m_{12}^2 s_{2\theta} & 0 & \Delta m_{12}^2 c_{2\theta} \end{pmatrix}\nu_l(z), \qquad (4)$$

where $L_e = \sqrt{2}G_F N_e$ ($L_n = -\sqrt{2}G_F N_n/2$) is the matter potential caused by the neutrino interaction with the matter electrons (neutrons), N_e (N_n) is the electron (neutron) density, $\Delta m_{12}^2 = m_2^2 - m_1^2$ and we have taken the liberty of omitting a term proportional to the unit matrix.

Now we assume that the neutrino flux flies into the field of a circularly polarized electromagnetic wave (CPW). It is convenient to choose the potential of the plane electromagnetic wave in the form

$$\mathbf{A}(\eta) = f(\mathbf{n}_x c_\eta + \xi \mathbf{n}_z s_\eta), \tag{5}$$

where the wave vector $\mathbf{k}$ is directed along a Cartesian y axis, f is the wave amplitude, η is the wave phase ($\eta = \omega t - ky$), ω is the wave frequency, and the Stokes parameter $\xi = \pm 1$ determines the CPW polarization (left- and right-handed, respectively).

$$\mathcal{L}_{em} = i\mu_{\nu_j\nu_k}\overline{\nu}_j\sigma_{\lambda\tau}F_{\lambda\tau}(1+\gamma_5)\nu_k + [a^L_{\nu_j\nu_k}\overline{\nu}_j\partial_\lambda F_{\lambda\tau}\gamma_\tau\gamma_5(1+\gamma_5)\nu_k + (L \to R)], \tag{6}$$

where $F_{\lambda\tau}$ is the electromagnetic field tensor, $\mu_{\nu_j\nu_k}$ and $a^{L,R}_{\nu_j\nu_k}$ are the matrix elements of the dipole and anapole moment.

Now the evolution of the flux consisting of the left-handed and right-handed neutrinos is defined by the following interaction Hamiltonian

$$\mathcal{H}^t_{int} = \mathcal{H}^{mat}_{int} + \mathcal{H}^{em}_{int}, \tag{7}$$

where

$$\mathcal{H}^{em}_{int} = \begin{pmatrix} a_{\nu_{eL}}d_z & \mu_{\nu_e}H_x & 0 & 0 \\ \mu_{\nu_e}H_x & -a_{\nu_{eR}}d_z & 0 & 0 \\ 0 & 0 & a_{\nu_{\mu L}}d_z & \mu_{\nu_e}H_x \\ 0 & 0 & \mu_{\nu_e}H_x & -a_{\nu_{\mu R}}d_z \end{pmatrix}, \tag{8}$$

$$\mathbf{d} = \text{rot } \mathbf{H} + \frac{\partial \boldsymbol{\mathcal{E}}}{\partial t},$$

and we have assumed that only diagonal elements of the multipole moments are non zero.

Here we shall concentrate on the ν_{eL} resonant conversions only. (i) $\nu_{eL} \to \nu_{eR}$ is a spin-flipping (SF) resonance, which is realized if the following condition

$$L_e + L_n + (a_{\nu_{eL}} + a_{\nu_{eR}})d_z = 0 \tag{9}$$

is fulfilled.

(ii)$\nu_{eL} \to \nu_{\mu L}$ is the MSW resonance. It takes place at the condition

$$2\Delta m_{12}^2 c_{2\theta} - 4E[L_e + (a_{\nu_{eL}} - a_{\nu_{eR}})d_z] = 0. \tag{10}$$

(iii) $\nu_{eL} \to \nu_{\mu R}$ is the spin-flavor flipping (SFF) resonance. The condition of its fulfilment is as follows

$$2\Delta m_{12}^2 c_{2\theta} - 4E[L_e + L_n + (a_{\nu_{eL}} + a_{\nu_{\mu R}})d_z] = 0. \tag{11}$$

Let us examine these resonant conversions under terrestrial conditions. When the neutrino flux is moving close to the Earth surface the matter potential has the size

$$L_e + L_n \approx \text{few} \times 10^{-17} \text{ eV}.$$

$$a_{\nu lL,R} = \frac{1}{6} < r^2 >_{lL,R} . \tag{12}$$

According to the up-to-date estimation $< r^2 >$ could be as large as 10^{-32} cm^2. Since Δm_{12}^2 has an order of 10^{-5} eV^2, then only the SF resonance could be observed. It is obvious that the interaction with the electric part of the CPW is more important. Using Eq. (9) we can find the values of the electric field strength and the frequency which could ensure the spin flip for the electron neutrino

$$\omega\mathcal{E} = -\xi \frac{c(L_e + L_n)}{e(a_{\nu_{eL}} + a_{\nu_{eR}})}, \tag{13}$$

and

$$\omega t = 2\pi n, \qquad n = 1, 2...$$

where for a while we have returned to the ordinary units. At present the laser fields can be as large as $\mathcal{E} \sim 10^{12}$ V/m. Making use of this value and substituting into Eq. (13) $a_{\nu eL} + a_{\nu eR} \sim 10^{-32}$ cm^2, we come to the following conclusion. In order to carry out the electron neutrino spin flip the frequency of the SPW ω should be of the order of 10^{15} s^{-1}. Such frequencies are available in modern lasers.

$$W_{\nu_{eL} \to \nu_{eR}} = \exp\left\{ -\frac{1}{\omega} \text{Im} \int_{y_r}^{y_0} \sqrt{\frac{A[y+B]^2 + C^2}{1-y^2}} dy \right\}, \tag{14}$$

where

$$A = \left[\frac{f(a_{\nu_{eL}} + a_{\nu_{eR}})(\mathbf{k}^2 + \omega^2)}{4}\right]^2 - \mu_{\nu_e}^2 f^2 \mathbf{k}^2, \qquad D = \frac{f\xi(L_e + L_n)(a_{\nu_{eL}} + a_{\nu_{eR}})(\mathbf{k}^2 + \omega^2)}{16},$$

$$B = A^{-1}D, \qquad C = A^{-1}\left[\frac{(L_e + L_n)^2}{16} + \mu_{\nu_e} f^2 \mathbf{k}^2 - A^{-1}D^2\right],$$

$$y_r = -\frac{\xi(L_e + L_n)}{f(a_{\nu_{eL}} + a_{\nu_{eR}})(\mathbf{k}^2 + \omega^2)}, \qquad y_0 = iC - B.$$

One can investigate the more general case when the left-handed and right-handed neutrino belonging to the same generation are mixed, i.e. they have different masses. Then, in two flavor approximation the connection between weak and mass eigenstates is as follows

$$\begin{pmatrix} \nu_{eL} \\ \nu_{eR} \\ \nu_{\mu L} \\ \nu_{\mu R} \end{pmatrix} = \mathcal{U} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \end{pmatrix}, \tag{15}$$

where

$$\mathcal{U} = \begin{pmatrix} c_\varphi & s_\varphi & 0 & 0 \\ -s_\varphi & c_\varphi & 0 & 0 \\ 0 & 0 & c_\varphi & s_\varphi \\ 0 & 0 & -s_\varphi & c_\varphi \end{pmatrix} \begin{pmatrix} c_\theta & 0 & s_\theta & 0 \\ 0 & c_\theta & 0 & s_\theta \\ -s_\theta & 0 & c_\theta & 0 \\ 0 & -s_\theta & 0 & c_\theta \end{pmatrix}. \tag{16}$$

where φ is the mixing angle between the left-handed and right-handed neutrino in l-generation, θ is the mixing angle between neutrinos belonging to different generations (for the sake of simplicity we assume that the mixing angles between left-handed and right-handed neutrino in every generation are equals and the same is true for the mixing angles between left-handed (right-handed) neutrinos belonging different generations).

The $\nu_{eL} \to \nu_{eR}$-resonance takes place under condition

$$-\Delta m_{12}^2 c_{2\theta} c_\varphi^2 - \Delta m_{34}^2 c_{2\theta} s_\varphi^2 + \Lambda c_{2\theta} + 4E(L_e + L_n + a_{\nu_{eL}} d_z) =$$
$$= -\Delta m_{12}^2 c_{2\theta} c_\varphi^2 - \Delta m_{34}^2 c_{2\theta} c_\varphi^2 - \Lambda c_{2\theta} - 4E a_{\nu_{eR}} d_z, \tag{17}$$

where

$$\Lambda = \frac{m_1^2 + m_2^2 - m_3^2 - m_4^2}{2}. \tag{18}$$

The MSW and SFF resonances for the left-handed electron neutrino occur under fulfilment of the conditions

$$-\Delta m_{12}^2 c_{2\theta} c_\varphi^2 - \Delta m_{34}^2 c_{2\theta} s_\varphi^2 + \Lambda c_{2\theta} + 4E(L_e + a_{\nu_{eL}} d_z) =$$
$$= -\Delta m_{12}^2 c_{2\theta} c_\varphi^2 - \Delta m_{34}^2 c_{2\theta} s_\varphi^2 + \Lambda c_{2\theta} + 4E a_{\nu_{\mu L}} d_z, \tag{19}$$
$$-\Delta m_{12}^2 c_{2\theta} c_\varphi^2 - \Delta m_{34}^2 c_{2\theta} c_\varphi^2 + \Lambda c_{2\theta} + 4E(L_e + L_n + a_{\nu_{eL}} d_z) =$$
$$= -\Delta m_{12}^2 c_{2\theta} s_\varphi^2 - \Delta m_{34}^2 c_{2\theta} s_\varphi^2 - \Lambda c_{2\theta}. \tag{20}$$

At present we have not any information about m_2, m_4 and φ. In all our investigation within the SM we are assuming that masses of the left-handed and right-handed neutrino are equal. So, it is not excluded that the MSW and SFF resonances could also be observable in the CPW fields available.

Let us briefly discuss the results obtained. The spin flip of the left-handed neutrino leads to its sterility. Analogously, the spin-flavor flip of ν_{eL} gives the same result. Neutrino telescopes (NT) practically do not detect neither ν_{eR} nor $\nu_{\mu R}$. So, if the neutrino flux has passed through the CPW field located before the NT and two above mentioned resonances have occurred, we will observe the increase of the flux. In the MSW resonance case the additional $\nu_{\mu L}$ neutrinos appear. But the modern NT's have different sensitivity with respect to the neutrinos belonging to different generations. For example, the sensitivity of Super-Kamiokande regards to the muon and tau neutrino is less than that regards to the electron neutrino by a factor of 6.

As far as the Majorana neutrino is concerned, the diagonal elements of the dipole magnetic moment is equal to zero. If the mixing scheme (2) is realized then only the

MSW and SFF resonances would be possible. However, the anapole interaction is too small compared with the quantity $\Delta m^2_{12} c_{2\theta}$ that it is extremely improbable to create the electromagnetic fields providing for the resonance conditions.

In summary, the possibility of the spin flip for the electron neutrino with the help of the CPW field open the comprehensive facilities. In the first place, we can measure or constrain the anapole neutrino moment. Secondly, we can ascertain the true mixing scheme in the neutrino sector. Thirdly, we can establish the neutrino nature within the definite model of the electroweak interaction.