Recent progress in the theoretical and experimental study of the neutron polarizabilities

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1. Introduction

The polarizability of a composite system is an elementary structure constant. It is well known that even a neutral system can interact with an external electromagnetic field if the system consists of charged constituents. For example, being put into a static electric field $\boldsymbol{E}$, a system consisting of two constituents with charges $+e$ and $-e$ acquires an induced dipole moment $\boldsymbol{d} = e\Delta\boldsymbol{r}$, where $\Delta\boldsymbol{r}$ is the displacement due to the field $\boldsymbol{E}$. One can show that the polarizability of such a system is

$$\alpha = 2\sum_{n>0} \frac{|<n\,|\,ez\,|\,0>|^2}{\varepsilon_n - \varepsilon_0},$$

where ε_n is the energy of the eigenstate $|\,n>$. In a few particular cases the summation can be performed analytically. For instance, in the case of the hydrogen atom one obtains a well known result

$$\alpha({}^1H) = \frac{9}{2}\, a_B^3,$$

where $a_B = 1/(e^2 m_e)$ is the Bohr radius.

Since, as we know, the nucleons have their internal structure (they consist of charged quarks), one can expect them to manifest this structure through the polarizability effect. And indeed, this is the case. The nucleons have their polarizabilities which characterize the first order response of internal structure of the nucleons to applied electric and magnetic fields. A deformation of the system's ground state caused by an incoming electromagnetic wave and encoded into electromagnetic polarizabilities of the system contributes to radiation of outgoing photons and thus shows itself in such observablies as the differential cross section of Compton scattering.

Let us consider as an example the low energy expansion of the spin averaged forward and backward Compton scattering amplitudes

$$T(\omega,0) = -\frac{Z^2e^2}{M_N}\mathbf{e}\cdot\mathbf{e}'^* + 4\pi(\alpha+\beta)\omega^2\mathbf{e}\cdot\mathbf{e}'^* + O(\omega^3),$$

$$T(\omega,180^0) = -\frac{Z^2e^2}{M_N}\mathbf{e}\cdot\mathbf{e}'^* + 4\pi(\alpha-\beta)\omega^2\mathbf{e}\cdot\mathbf{e}'^* + O(\omega^3),$$

where Z and M_N is the charge ($Z_p = 1$ and $Z_n = 0$) and mass of the nucleon, respectively, $\mathbf{e}$ and $\mathbf{e}'$ being the polarizations of the initial and final photons. An inspection of the above relations shows that the amplitude at forward and backward angles T depends on the sum and the difference of the polarizabilities, respectively. The sum of the polarizabilities is usually obtained in an indirect way making the use of predictions of the Baldin sum rule. This sum rule relates the sum $\alpha_N + \beta_N$ with the integral over the photoabsorption cross section $\sigma_N(\omega)$:

$$\alpha_N + \beta_N = \frac{1}{2\pi^2}\int_{\omega_0}^{\infty} \frac{\sigma_N(\omega)}{\omega^2}\,d\omega,$$

where ω_0 is photoabsorption threshold. Due to the factor ω^{-2} in the integrand, this integral converges very rapidly and it can be evaluated rather reliably. A recent reevaluation of this sum rule [*M.I. Levchuk, A.I. L'vov, Nucl. Phys.*, ***A674,*** *335 (2000)*] gives quite precise values for the sum of the nucleon polarizabilities

$$\alpha_p + \beta_p = 14.0 \pm 0.3, \qquad \alpha_n + \beta_n = 15.2 \pm 0.5,$$

in units of $10^{-4}\,\mathrm{fm}^3$ which will be used for the polarizabilities. Therefore, in order to have α_N and β_N separately, one needs to measure the difference of the polarizabilities and this can be done by measurements of observables in nucleon Compton scattering at backward angles.

The absence of a free dense neutron target makes it impossible to measure the neutron polarizabilities in free neutron Compton scattering experiments. Almost fifty years ago, electromagnetic scattering of low-energy neutrons in the electric fields of heavy nuclei was firstly used to extract the electric neutron polarizability α_N [*R.M. Thaler, Phys. Rev.* ***114***, *827 (1959)*]. Later on there were many attempts of measuring α_n along this method:

L. Koester, W. Waschkowsky, and J. Meier, Z. Phys. ***A329***, *229 (1988);*

J. Schmiedmayer, H. Rauch, and P. Riehs, Phys. Rev. Lett. ***61***, *1065 (1988);* ***61***, *2509 (1988);*

J. Schmiedmayer et al., Phys. Rev. Lett. ***66***, *1015 (1991);*

Yu.A. Aleksandrov, Fundamental properties of the neutron (Clarendon Press, Oxford, 1992);

L. Koester et al., Phys. Rev. ***C51***, *3363 (1995).*

However, the final conclusion on the value of α_n remained unclear. For example, a result of an Oak Ridge experiment, $\alpha_n = 12.0 \pm 1.5 \pm 2.0$, contradicts the Munich value $\alpha_n = 0 \pm 5$. Moreover, there is an argument [*T.L. Enik et al., Sov. J. Nucl. Phys.* ***60***, *567 (1997)*] that the systematic (in fact, theoretical) uncertainty, which is a very delicate problem for those experiments, might be strongly underestimated in the Oak Ridge experiment so that its result possibly might be quoted as $7 \le \alpha_n \le 19$. Note that the method does not constrain the magnetic polarizability β_n at all.

Scattering real photons off neutrons bound in nuclei is another option for measuring the neutron polarizabilities. Of course, such experimental studies of neutron Compton scattering and a further extraction of the neutron polarizabilities are much more difficult than those for the proton. First, because of the absence of dense free-neutron targets, actual measurements of γn-scattering are forced to have a deal with neutrons bound in nuclei and hence to take into account effects of the nuclear environment. Second, due to vanishing the neutron Thomson scattering amplitude (viz. the amplitude of photon scattering off the electric charge of the neutron which is zero), the contribution of polarizabilities of the neutron to the differential cross section at low energies ($\le$100 MeV) turns out to be rather small.

To minimize theoretical uncertainties in the interpretation of experimental data, preference should be given to the use of the deuteron as a "neutron target". Depending on the final np-state, two reactions, $\gamma d \to \gamma np$ and $\gamma d \to \gamma d$, can be considered.

2. Exclusive Compton scattering on the deuteron $\gamma d \to \gamma np$

A suggestion to exploit the reaction $\gamma d \to \gamma np$ in the neutron quasi-free kinematics region for study of Compton scattering on the neutron was made in Ref. [*M.I. Levchuk, A.I. L'vov, and V.A. Petrun'kin, Few-Body Syst.* ***16****, 101 (1994)*].

An idea of this method is rather simple. One can estimate from the value of the deuteron binding energy, $E_b = 2.2246$ MeV, that the characteristic momenta of the nucleons in the deuteron are about 50 MeV/c, i.e. the nucleons are practically at rest inside the deuteron. Let us suppose that after photon scattering off the deuteron, we observe, in the final state, the neutron with high momentum and the proton with low momentum. Such a situation means that probably the photon was scattered by the neutron rather them by the proton. The corresponding mechanisms are depicted in Fig. 1*a*.

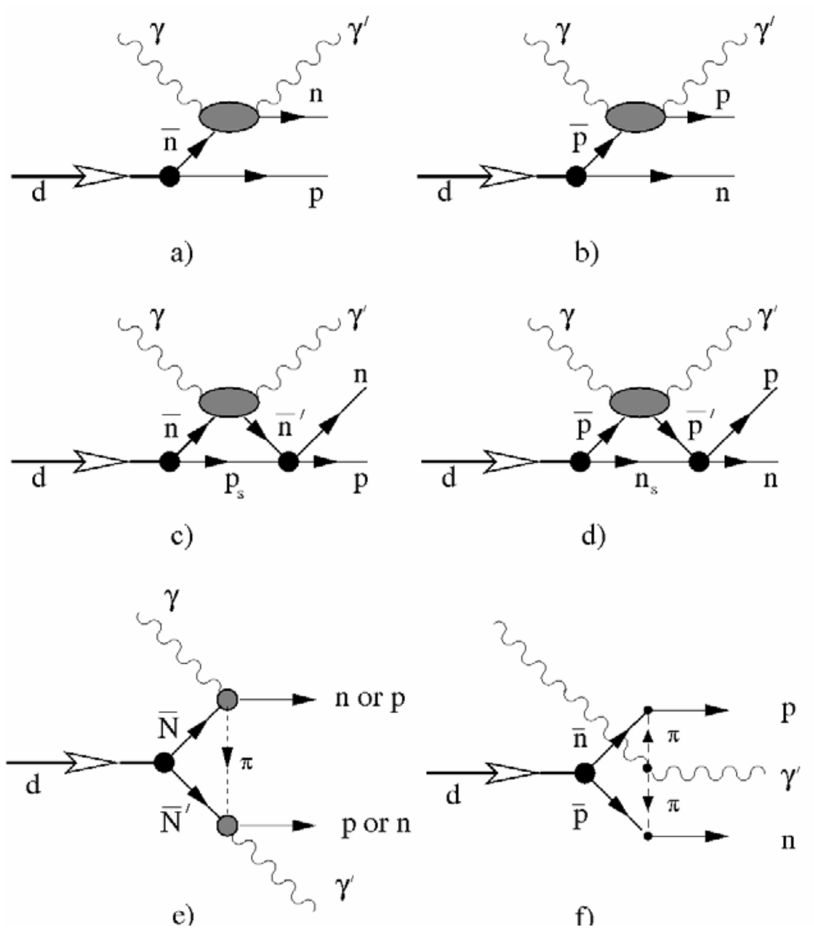

Fig. 1. Main graphs contributing to the reaction
$\gamma d \to \gamma' np$.

The kinematics with the low proton momenta in the final state is usually called as "neutron quasi-free kinematics" and the proton in such a kinematics is referred to as a spectator.

Of course, there are other mechanisms which have to be taken into account. Diagrams *a* and *b* in Fig. 1 correspond to the case when the final nucleons after photon scattering spread as a plane wave. But the situation is possible when these nucleons interact with each other in the final state (diagrams *c* and *d* in Fig. 1). It is known that such an interaction is very strong at low relative momentum of the final nucleons.

One more mechanism giving rise to the reaction amplitude is shown in diagrams *e* and *f* in Fig. 1. It is often referred to as meson-exchange seagulls (MES). Physically its origin is rather evident. We know that nuclear forces are due to exchanges by mesons (e.g., π, ρ, ω and so on). Interaction of the initial and final photons with these mesons is just responsible for the mechanism above. Unlike the pole diagrams *a* and *b* and these with final NN-interaction, MES, as a rule, do not manifest themselves in the cross section via peak structures only form smooth background.

A big advantage of using the method above in order to investigate the neutron Compton scattering amplitude is that it can be carefully tested for the proton case by comparing measured quasi-free proton cross sections with available free data. Such a test was performed in an experiment at MAMI-B [*F. Wissmann et al., Nucl. Phys.* ***A660****, 232 (1999)*] in which the energy dependence of the differential cross section of the reaction $\gamma d \to \gamma' np$ was measured.

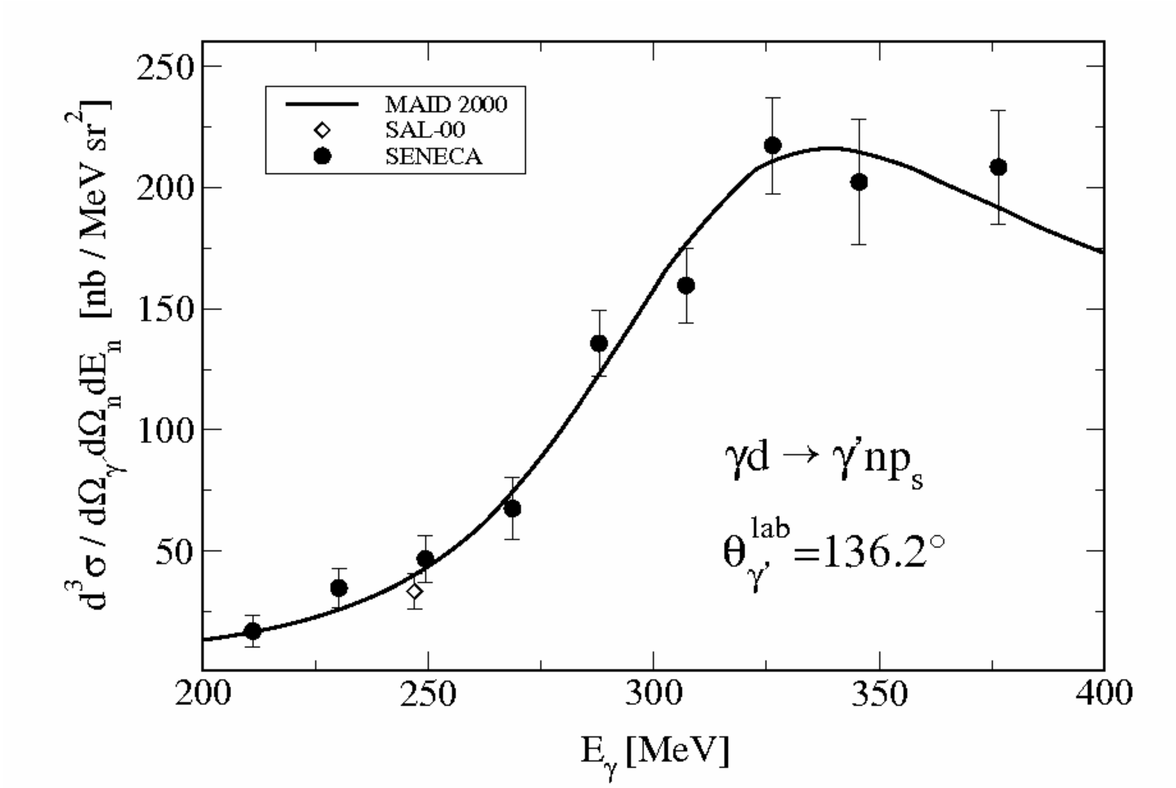

Fig. 2. Triple differential cross section of the reaction $\gamma d \to \gamma' np$ in the center of PQFP at $\theta_{\gamma'}^{\text{lab}} = 136.2^{\text{o}}$. Filled circles: MAMI-B experiment and diamond: SAL experiment [*N.R. Kolb et al., Phys. Rev. Lett.* ***85****, 1388 (2000)*].

The values for the proton polarizabilities were taken those as they were found in experiments on free proton targets. One can see from Fig. 2 that the agreement between predicted cross sections and measured ones is very good. Therefore, there are all grounds for believing that this method can be applicable in the neutron case as well.

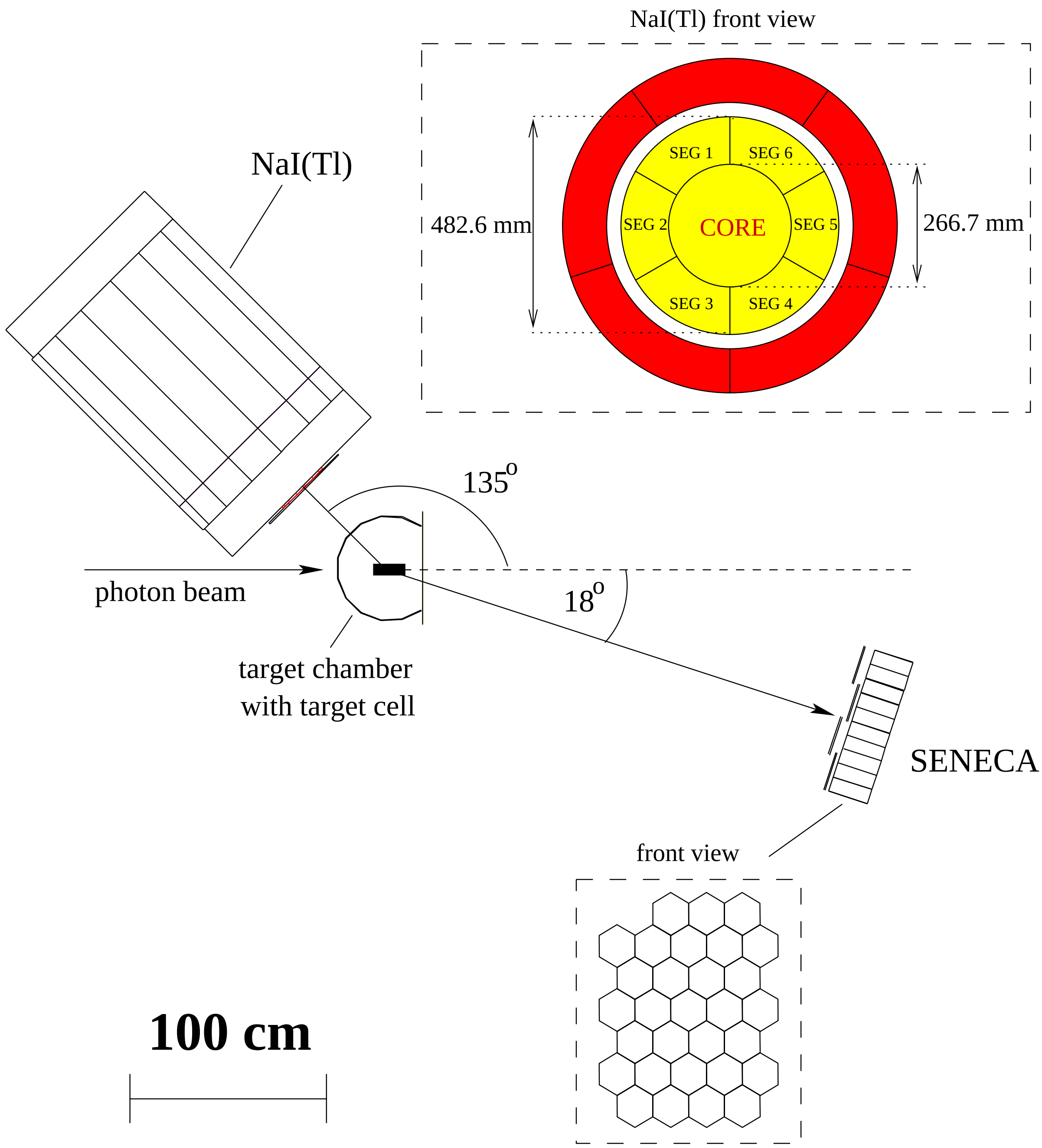

Fig. 3. The MAMI-B experimental setup used to measure quasi-free Compton scattering from the bound neutron and proton. The scattered photons were detected with the large volume NaI(Tl) detector, the recoiling neutrons and protons with the SENECA detector system. Liquid deuterium and liquid hydrogen have been used as target materials.

K. Kossert et al., Phys. Rev. Lett. ***88****, 162301 (2002).*
K. Kossert et al., Eur. Phys. J. ***A16****, 259 (2003).*

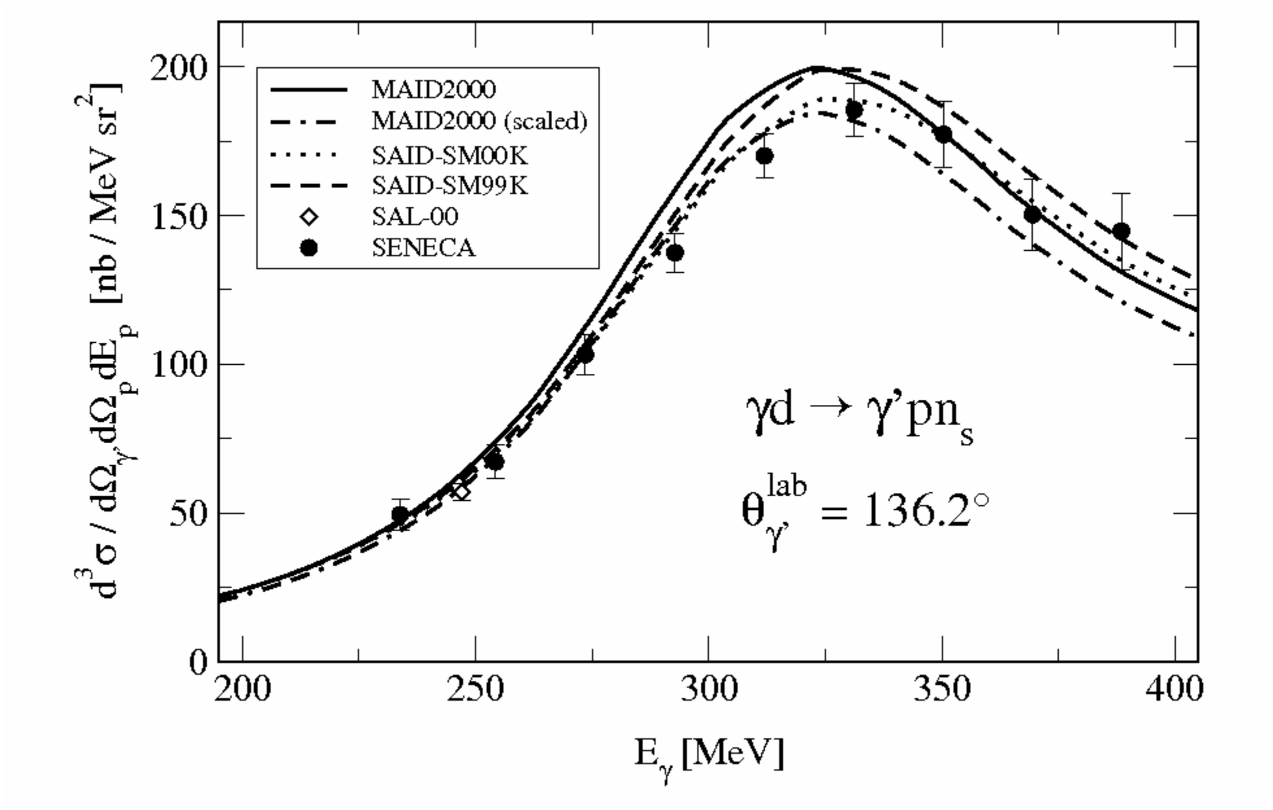

Fig. 4. Triple differential cross section of the reaction $\gamma d \rightarrow \gamma' np$ in the center of NQFP at $\theta^{\rm lab}_{\gamma'} = 136.2^{\rm o}$. Filled circles: MAMI-B experiment and diamond: SAL experiment. Only statistical errors are shown

The values for the neutron polarizabilities found in the experiment are as follows

$$\alpha_n = 12.5 \pm 1.8({\rm stat})^{+1.1}_{-0.6}({\rm syst}) \pm 1.1({\rm model}),$$
$$\beta_n = 2.7 \mp 1.8({\rm stat})^{+0.6}_{-1.1}({\rm syst}) \mp 1.1({\rm model}).$$

The anticorrelated errors are due to the application of the Baldin sum rule result.

Despite the high precision of α_n and β_n, further confirmations of these values with other methods would be of great importance.

3. Compton scattering off the deuteron.

Elastic photon, or Compton, scattering off the deuteron provides third method for determining the neutron polarizabilities which was mentioned already in Ref. [*A.M. Baldin, Nucl. Phys.* ***18****, 310 (1960)*] forty seven years ago and later on has been discussed in many papers

N.V. Maksimenko, Ph.D. thesis, Institute of Physics, Minsk (1973);
T. Wilbois, P. Wilhelm and H. Arenhövel, Few-Body Syst. Suppl. ***9****, 263 (1995);*
M.I. Levchuk and A.I. L'vov, Few-Body Syst. Suppl. ***9****, 439 (1995);*
J.J. Karakowski and G.A. Miller, Phys. Rev. ***C60****, 014001 (1999);*
J.-W. Chen, Nucl. Phys. ***A653****, 375 (1999);*
S.R. Beane et al., Nucl. Phys. ***A656****, 367 (1999);*
M.I. Levchuk and L'vov, Nucl. Phys., ***A674,*** *335 (2000).*

The presence of the proton next to the neutron and the coherence of the proton and neutron contributions makes two advantages. First, the $O(\omega^2)$ contribution of the neutron polarizabilities to the scattering amplitude can interfere with the $O(1)$ contribution from proton Thomson scattering, so that a sensitivity of the differential cross section with respect to the polarizabilities is enhanced. Of course, only the isospin averaged nucleon polarizabilities, $\alpha_N = (\alpha_p + \alpha_n)/2$ and $\beta_N = (\beta_p + \beta_n)/2$, can be measured with this method. But that is not a major problem since the proton values are quite accurate Nevertheless, various binding corrections, including meson-exchange currents (MEC) and meson-exchange seagulls (MES), are rather important and have to be introduced and carefully evaluated.

Two types diagram contribute to the scattering amplitude. First of them is the so-called resonance diagrams which are shown in Fig. 5*a* and *b*.

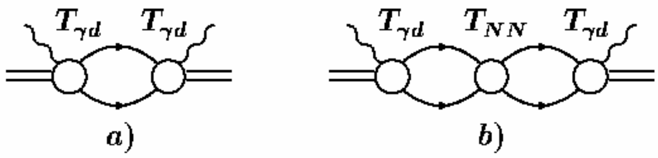

Fig. 5. The resonance contributions. Shown are terms without (*a*) and with NN-rescattering in the intermediate state (*b*).

In order to calculate the resonance amplitude $R(E_\gamma, \Theta_\gamma)$, we introduce the off-shell T-matrix of NN-scattering, $T_{NN}(E)$, and write the NN-propagator $G(E) = (E - H + i0)^{-1}$ in the form

$$G(E) = G_0(E) + G_0(E) T_{NN}(E) G_0(E),$$

where $G_0(E) = (E - H_0 + i0)^{-1}$ is the propagator of free nucleons. Then $R(E_\gamma, \Theta_\gamma)$ turns out to be the sum of two terms, without and with NN-rescattering in the intermediate state.

Diagrammatic representation of the seagull contributions is as follows

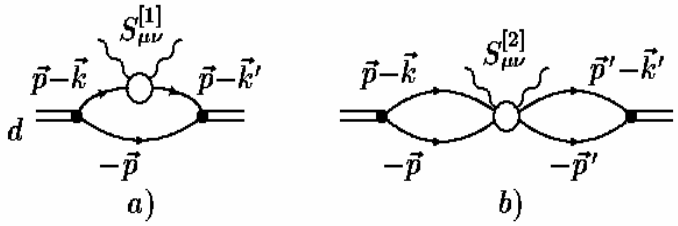

Fig. 6. Shown are the one-body (a) and two-body (b) seagull amplitudes of γd scattering.

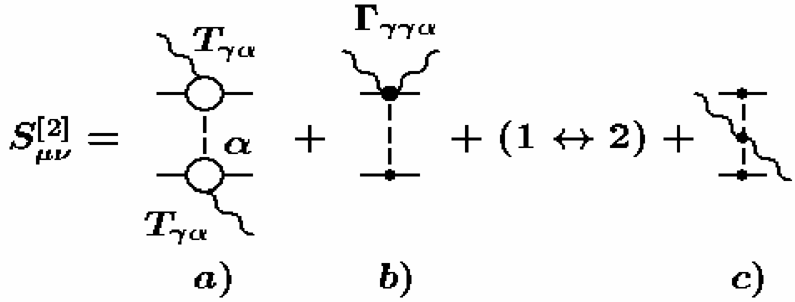

Fig. 7. The diagrammatic representation of the two-body seagull $S^{[2]}$. The symbol α stands for different types of mesons. $T_{\gamma\alpha}$ means the meson photoproduction amplitude.

The meaning for the vertex $\Gamma_{\gamma\gamma\alpha}$ is explained in Fig. 8.

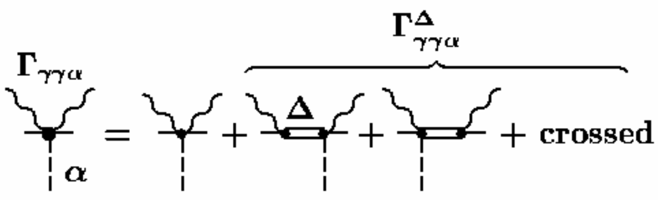

Fig. 8. The diagrammatic representation of the vertex $\Gamma_{\gamma\gamma\alpha}$.

The symbol Δ means the contribution of the Δ-isobar.

Using tagged photons with energies of 49 and 69 MeV the Illinois group [*M.A. Lucas, Ph.D. Thesis, University of Illinois (1994)*] obtained the first data on the differential cross section which were precise enough to reveal the effect from the nucleon polarizabilities. Five new data points were obtained later on by the SAL group [*D.L. Hornidge et al., Phys. Rev. Lett.* ***84****, 2334 (2000)*] at about 94 MeV of the incident photon energy. A bulk of experimental data has been recently provided in an experiment at MAX-lab (Lund, Sweden) [*M. Lundin et al., Phys. Rev. Lett.* ***90****, 192501 (2003)*]. All the above data are shown in Fig. 8 together with the predictions from Refs. [*LevLv95,00*].

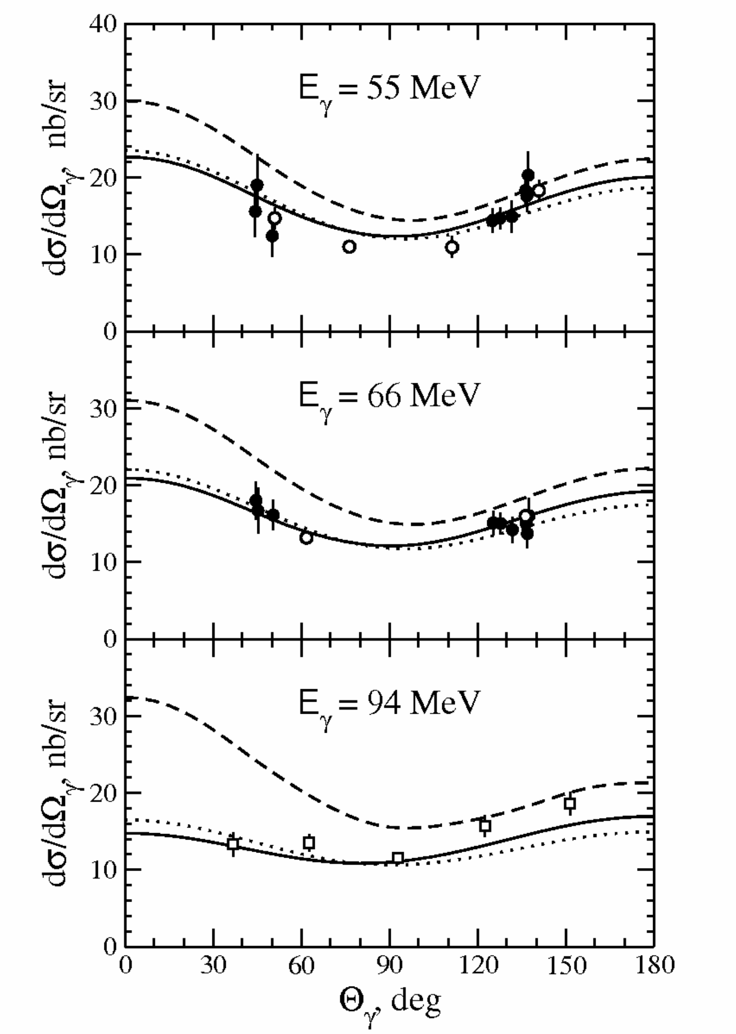

Fig. 8. CM differential cross sections for Compton scattering off the deuteron. Open circles: Illinois results; open squares: SAL results; filled circles: Lund experiment. Curves are predictions from Refs. [*LevLv95,00*]. Resonance diagrams: dashed curves; one-body and two-body seagull (without rescattering): dotted curves: rescattering is included in solid curves.

If all these data are fitted using the theoretical model of Refs. [*LevLv95,00*], the following ”global average” is inferred:

$$\alpha_N + \beta_N = 16.7 \pm 1.6, \quad \alpha_N - \beta_N = 4.8 \pm 2.0,$$

with $\chi^2/N_{\text{dof}} = 38/(29-2)$. Using these values and the

Baldin sum rule predictions, one obtains the following values for the neutron polarizabilities extracted from the experiments on elastic photon scattering off the deuteron

$$\alpha_n = 7.2 \pm 2.1, \quad \beta_n = 8.1 \mp 2.1.$$

Here both statistical and systematic errors are combined together, the latter being taken into account through a re-scaling of measured cross sections within their normalization uncertainties. As to the model uncertainty in the derived values, we estimate them as about ±3.0.

Combining the values obtained in deuteron Compton scattering with these extracted from the quasi-free neutron data and neutron transmission experiments, the PDG group has reported the presently most reliable values for the neutron polarizabilities [*W.-M. Yao et al., Review of Particle Physics, J. Phys.*, ***C 33***, *1 (2006)*]:

$$\alpha_n = 11.4 \pm 1.5, \quad \beta_n = 3.7 \pm 2.0.$$

n ELECTRIC POLARIZABILITY α_n

Following is the electric polarizability α_n defined in terms of the induced electric dipole moment by $\mathbf{D} = 4\pi\epsilon_0\alpha_n\mathbf{E}$. For a review, see SCHMIEDMAYER 89.

VALUE (10^{-3} fm^3)	*DOCUMENT ID*		*TECN*	*COMMENT*
$\mathbf{0.98^{+0.19}_{-0.23}}$ **OUR AVERAGE**	Error includes scale factor of 1.1.			
0.0 ± 0.5	[14] KOESTER	95	CNTR	n Pb, n Bi transmission
$1.20 \pm 0.15 \pm 0.20$	SCHMIEDM...	91	CNTR	n Pb transmission
$1.07^{+0.33}_{-1.07}$	ROSE	90B	CNTR	$\gamma d \rightarrow \gamma n p$
0.8 ± 1.0	KOESTER	88	CNTR	n Pb, n Bi transmission
1.2 ± 1.0	SCHMIEDM...	88	CNTR	n Pb, n C transmission
• • • We do not use the following data for averages, fits, limits, etc. • • •				
$1.17^{+0.43}_{-1.17}$	ROSE	90	CNTR	See ROSE 90B

[14] KOESTER 95 uses natural Pb and the isotopes 208, 207, and 206. See this paper for a discussion of methods used by various groups to extract α_n from data.

Citation: W.-M. Yao *et al.* (Particle Data Group), J. Phys. G **33**, 1 (2006) (URL: http://pdg.lbl.gov)

n ELECTRIC POLARIZABILITY α_n

Following is the electric polarizability α_n defined in terms of the induced electric dipole moment by $\mathbf{D} = 4\pi\epsilon_0\alpha_n\mathbf{E}$. For a review, see SCHMIEDMAYER 89.

VALUE (10^{-4} fm^3)	*DOCUMENT ID*		*TECN*	*COMMENT*
11.6± 1.5 OUR AVERAGE				
$12.5\pm 1.8^{+1.6}_{-1.3}$	[15] KOSSERT	03	CNTR	$\gamma d \to \gamma p n$
$8.8\pm 2.4\pm 3.0$	[16] LUNDIN	03	CNTR	$\gamma d \to \gamma d$
$12.0\pm 1.5\pm 2.0$	SCHMIEDM...	91	CNTR	*n* Pb transmission
$10.7^{+\ 3.3}_{-10.7}$	ROSE	90B	CNTR	$\gamma d \to \gamma n p$
• • • We do not use the following data for averages, fits, limits, etc. • • •				
13.6	[17] KOLB	00	CNTR	$\gamma d \to \gamma n p$
0.0 ± 5.0	[18] KOESTER	95	CNTR	*n* Pb, *n* Bi transmission
$11.7^{+\ 4.3}_{-11.7}$	ROSE	90	CNTR	See ROSE 90B
$8\ \pm 10$	KOESTER	88	CNTR	*n* Pb, *n* Bi transmission
$12\ \pm 10$	SCHMIEDM...	88	CNTR	*n* Pb, *n* C transmission

[15] KOSSERT 03 gets $\alpha_n - \beta_n = (9.8 \pm 3.6^{+2.1}_{-1.1} \pm 2.2) \times 10^{-4}$ fm^3, and uses $\alpha_n + \beta_n = (15.2 \pm 0.5) \times 10^{-4}$ fm^3 from LEVCHUK 00. Thus the errors on α_n and β_n are anti-correlated.

[16] LUNDIN 03 measures $\alpha_N - \beta_N = (6.4 \pm 2.4) \times 10^{-4}$ fm^3 and uses accurate values for α_p and α_p and a precise sum-rule result for $\alpha_n + \beta_n$. The second error is a model uncertainty, and errors on α_n and β_n are anticorrelated.

[17] KOLB 00 obtains this value with a lower limit of 7.6×10^{-4} fm^3 but no upper limit from this experiment alone. Combined with results of ROSE 90, the 1-σ range is (7.6–14.0) $\times 10^{-4}$ fm^3.

[18] KOESTER 95 uses natural Pb and the isotopes 208, 207, and 206. See this paper for a discussion of methods used by various groups to extract α_n from data.

n MAGNETIC POLARIZABILITY β_n

VALUE (10^{-4} fm^3)	*DOCUMENT ID*		*TECN*	*COMMENT*
3.7±2.0 OUR AVERAGE				
$2.7\pm 1.8^{+1.3}_{-1.6}$	[19] KOSSERT	03	CNTR	$\gamma d \to \gamma p n$
$6.5\pm 2.4\pm 3.0$	[20] LUNDIN	03	CNTR	$\gamma d \to \gamma d$
• • • We do not use the following data for averages, fits, limits, etc. • • •				
1.6	[21] KOLB	00	CNTR	$\gamma d \to \gamma n p$

[19] KOSSERT 03 gets $\alpha_n - \beta_n = (9.8 \pm 3.6^{+2.1}_{-1.1} \pm 2.2) \times 10^{-4}$ fm^3, and uses $\alpha_n + \beta_n = (15.2 \pm 0.5) \times 10^{-4}$ fm^3 from LEVCHUK 00. Thus the errors on α_n and β_n are anti-correlated.

[20] LUNDIN 03 measures $\alpha_N - \beta_N = (6.4 \pm 2.4) \times 10^{-4}$ fm^3 and uses accurate values for α_p and α_p and a precise sum-rule result for $\alpha_n + \beta_n$. The second error is a model uncertainty, and errors on α_n and β_n are anticorrelated.

[21] KOLB 00 obtains this value with an upper limit of 7.6×10^{-4} fm^3 but no lower limit from this experiment alone. Combined with results of ROSE 90, the 1-σ range is (1.2–7.6) $\times 10^{-4}$ fm^3.

Therefore, one can conclude that the proposed methods to measure the neutron polarizabilities in the reactions $\gamma d \to \gamma' np$ and $\gamma d \to \gamma d$ have allowed one to extract the values of the polarizabilities with the notably better accuracy in comparison with that achieved in neutron transmission experiments for more than 50 years of the measurements. We hope that approved experiments in Darmchtadt (Germany) and at MAX-lab on measurements of the differential cross section of the reaction $\gamma d \to \gamma d$ will provide further improvement of the accuracy for values of the neutron polarizabilities.