Moving medium electrodynamics approach for description of the static polarizable properties of the nucleons and its application for VVCS amplitude

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Introduction

At present time, the description of the Compton scattering off hadrons is performed by nonrelativistic Hamiltonian function
[S. Scherer, A.Yu. Korchin, J.H. Koch, Phys. Rev. C54, 904 (1996)],
[D. Babusci, J. Jiordano, A.J. L'vov, J. Matone, A.N.Nathan, Phys. Rev. C58, 1013 (1998).]

However for the extraction of the more essential experimental and theoretical information about hadron polarizabilities it is necessary to develop the Lagrangian of electromagnetic field interaction with polarizable particle in covariant form.

Introduction

The such Lagrangian was constructed by the phenomenological formfactor-based approach [A.I. L'vov, Int. J. Mod. Phys. A., 145 (1993)]. In the present report for the construction of the similar Lagrangian the developed in [N.V. Maksimenko, L.G. Moroz (D2-11707) Dubna, 533] approach is used in more consistently and completely way that allows us to determine not only the Lagrangian itself but the energy-momentum tensor of electromagnetic field interaction with polarizable particles.

Introduction

This approach allows one to take into account introduced in

[N.V. Maksimenko, L.G. Moroz (D2-11707) Dubna, 533]

[M.I.Levchuk, L.G.Moroz, Vesti AN BSSR Ser.: fiz.-mat. nauk 1. 49 (1985)]

[S. Ragusa, Phys. Rev. **D47**, 3757 (1993).]

spin polarizabilities of hadrons.

Introduction

One of the most interesting topic in Compton scattering investigation is the measurement of Q^2-dependence of the forward polarizabilities in deep-inelastic scattering (DIS)
[Jefferson Lab E94010 and E97110 Collaborations]
that can be presented as the imaginary part of VVCS.
[D. Drechsel, B. Pasquini, M. Vanderhaeghen, Phys. Rept. **378**, 99 (2003)]

Introduction

Performing a such kind of the experiments it will be also important to take into account radiative effects correctly. Notice that FORTRAN codes have been already developed for estimation of the radiative effects from lepton legs in elastic (MASCARAD) and deep-inelastic (POLRAD, ESFRAD) scattering. Basing on these codes Monte Carlo generators for simulation of radiative events in elastic (ELRADGEN) and deep-inelastic (RADGEN) scattering was developed. All of these codes are available on http://www.jlab.org/RC.

Introduction

Another interesting source of radiative effect contribution to the imaginary part of VVCS amplitude is photon emission from nucleon line. On the lowest order this process can be presented in the frame of the static polarizabilities. Using the standard Feynman technique on the lowest order this process can be presented in the frame of the static polarizability contributions.

Lagrangian

The Lagrangian we present in a following way:

$$\mathcal{L}_I = -j_\mu{}^{(M)}(x) A^\mu(x),$$

where $\partial^\mu j_\mu{}^{(M)} = 0,\ j_\mu{}^{(M)} = -\partial^\rho G^I_{\rho\mu},\ G^I_{\rho\mu} = -G^I_{\mu\rho}$

and $G^I_{\mu\nu} = U_\mu D_\nu - U_\nu D_\mu + \varepsilon_{\mu\nu\rho\sigma} U^\rho M^\sigma,\ U = \frac{1}{\sqrt{1-\mathbf{v}^2}}\{1, \mathbf{v}\},$

here D_ν (M^σ) electric (magnetic) dipole moment of the system. As a result

$$\mathcal{L}_I = -\frac{1}{2}(D_\mu e^\mu - M_\mu h^\mu),$$

where $e^\mu = F^{\mu\nu} U_\nu, h^\mu = \widetilde{F}^{\mu\nu} U_\nu,$

$$D_\mu = 4\pi\alpha'_0 g_{\mu\nu} e^\nu,\ M_\mu = 4\pi\beta'_0 g_{\mu\nu} h^\nu.$$

Lagrangian

After some transformation the Lagrangian can be present as:

$$\mathcal{L}_0 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu},\ \mathcal{L}_I = -2\pi\left[(\alpha_0' - \beta_0')\ e^2 + \frac{\beta_0'}{2}F^2\right].$$

From the Eiler-Lagrange equation of motion for media in rest the definitions of charge and current densities for bound charges are follow

$$\rho^{(M)} = -4\pi\alpha_0'\ \mathrm{div}\ \mathbf{E},\ \mathbf{j}^{(M)} = 4\pi(\alpha_0'\partial_t\mathbf{E} - \beta_0'\mathrm{rot}\ \mathbf{H}),$$

The energy-momentum tensor

From the Noether's theorem using our Lagrangian one can find that canonical energy-momentum tensor

$$T^{\mu\nu} = -G^{\mu\rho}(\partial^{\nu} A_{\rho}) - g^{\mu\nu}\frac{1}{4}(F_{\rho\sigma}G^{\rho\sigma}).$$

we determine the metric energy-momentum tensor as

$$\widetilde{T}^{\mu\nu} = F_{\rho}{}^{\nu}G^{\mu\rho} + \frac{1}{4}g^{\mu\nu}(F_{\rho\sigma}G^{\rho\sigma}).$$

The energy density for media in rest

$$\widetilde{T}^{00} = \omega = \frac{1}{2}(\varepsilon\mathbf{E}^2 + \mu\mathbf{H}^2).$$

The correspondence principle

The electric and magnetic polarizabilities extracted from our Lagrangian can be presented in the form:

$$\mathcal{L}_I = -2\pi(\alpha_0' F_{\mu\rho} F_\sigma{}^\mu - \beta_0' \widetilde{F}_{\mu\rho} \widetilde{F}_\sigma{}^\mu) U^\rho U^\sigma, \ \widetilde{F}_{\mu\rho} = \frac{1}{2}\varepsilon_{\mu\rho\sigma\kappa} F^{\sigma\kappa}.$$

Using the correspondence principle between classical mechanics and quantum field theory we find:

$$\mathcal{L}_I = \frac{2\pi}{M}\left[\alpha_0 F_{\mu\rho} F_\sigma{}^\mu - \beta_0 \widetilde{F}_{\mu\rho} \widetilde{F}_\sigma{}^\mu\right] \widetilde{\Theta}^{\rho\sigma}, \ \alpha_0' = \rho\alpha_0, \ \beta_0' = \rho\beta_0,$$

where ρ is induced dipole momentum density, $\widetilde{\Theta}^{\rho\sigma} = \frac{1}{2}(\Theta^{\rho\sigma} + \Theta^{\sigma\rho})$ is the energy-momentum tensor of spinor field and: $\Theta^{\rho\sigma} = \frac{i}{2}\overline{\psi}\gamma^\rho \overleftrightarrow{\partial^\sigma} \psi.$

The energy-momentum tensor

The Lagrangian: $\mathcal{L}_I = -\frac{1}{4}F_{\mu\nu}G^{(S)I\mu\nu}$ with

$$G^{(S)I\mu\nu} = -\frac{4\pi}{M}\{(\alpha_0-\beta_0)[F^{\mu\sigma}\widetilde{\Theta}_\sigma{}^{\nu} - F^{\nu\sigma}\widetilde{\Theta}_\sigma{}^{\mu}]+\beta_0\widetilde{\Theta}_\rho{}^{\rho}F^{\mu\nu}\}$$

allow us to define the metric momentum-energy tensor for interaction:

$$\widetilde{T}_I^{\mu\nu} = F_\rho{}^{\nu}G_I^{(S)\mu\rho} + \frac{1}{4}g^{\mu\nu}(F_{\rho\sigma}G^{(S)I\rho\sigma}).$$

For the particle in a rest its interaction energy reads [D. Babusci, J. Jiordano, A.J. L'vov, J. Matone, A.N.Nathan, Phys. Rev. C58, 1013 (1998).]

$$H_I = \int d^3x T^{00} = -2\pi(\alpha_0\mathbf{E}^2 + \beta_0\mathbf{H}^2).$$

Static polarizability vertex

If we will follow the notations of Appendix B of [T.-P. Cheng, L.-F. Li, Gauge theory of elementary particle physics, Oxford (1984)] our Lagrangian can be presented as:

$$\mathcal{L}^{pol}_{eff} = \int \prod_{i=1}^{4} \left[d^4x_i \delta^4(x - x_i)\right] \alpha^{r'r}_{\sigma\delta}(x_1, x_2, x_3, x_4) \times \bar{\psi}_{r'}(x_3)\psi_r(x_1)A^{\sigma}(x_4)A^{\delta}(x_2),$$

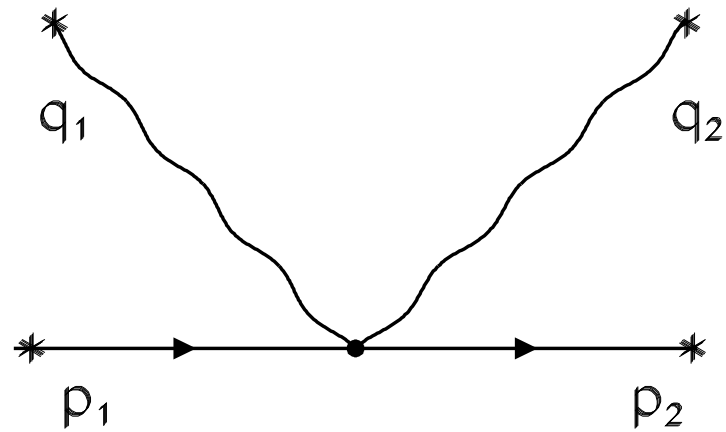
where

$$\alpha^{r'r}_{\sigma\delta}(x_1, x_2, x_3, x_4) = \int \frac{d^4p_1}{(2\pi)^4}\frac{d^4p_2}{(2\pi)^4}\frac{d^4q_1}{(2\pi)^4}\frac{d^4q_2}{(2\pi)^4} \times \tilde{\alpha}^{r'r}_{\sigma\delta}(p_1, q_1, p_2, q_2) e^{ip_1(x-x_1)+iq_1(x-x_2)-ip_2(x-x_3)-iq_2(x-x_4)},$$

Here p_1 and q_1 (p_2 and q_2) are the incoming (outgoing) nucleon and photon momenta respectively.

Static polarizability vertex

Polarizability vertex



After substitution:

$$\partial/\partial x_1 \to ip_1,\ \partial/\partial x_2 \to iq_1,\ \partial/\partial x_3 \to -ip_2,\ \partial/\partial x_4 \to -iq_2$$

we found

$$\tilde{\alpha}^{r'r}_{\sigma\delta}(p_1,q_1,p_2,q_2) = -\frac{\pi}{M}(p_1^\nu + p_2^\nu)\gamma_\mu^{r'r}\left((\alpha_0-\beta_0)[q_2^\mu\delta_\sigma^\rho - q_2^\rho\delta_\sigma^\mu]\right.$$

$$\left.[q_{1\rho}g_{\nu\delta} - q_{1\nu}g_{\rho\delta}] - 1/2\delta_\nu^\mu\beta_0\left[q_2^\rho\delta_\sigma^\gamma - q_2^\gamma\delta_\sigma^\rho\right]\left[q_{1\rho}g_{\gamma\delta} - q_{1\gamma}g_{\rho\delta}\right]\right)$$

Static polarizability vertex

According crossing symmetry $q_1 \leftrightarrow -q_2$ we receive the final expression for polarizability vertex as:

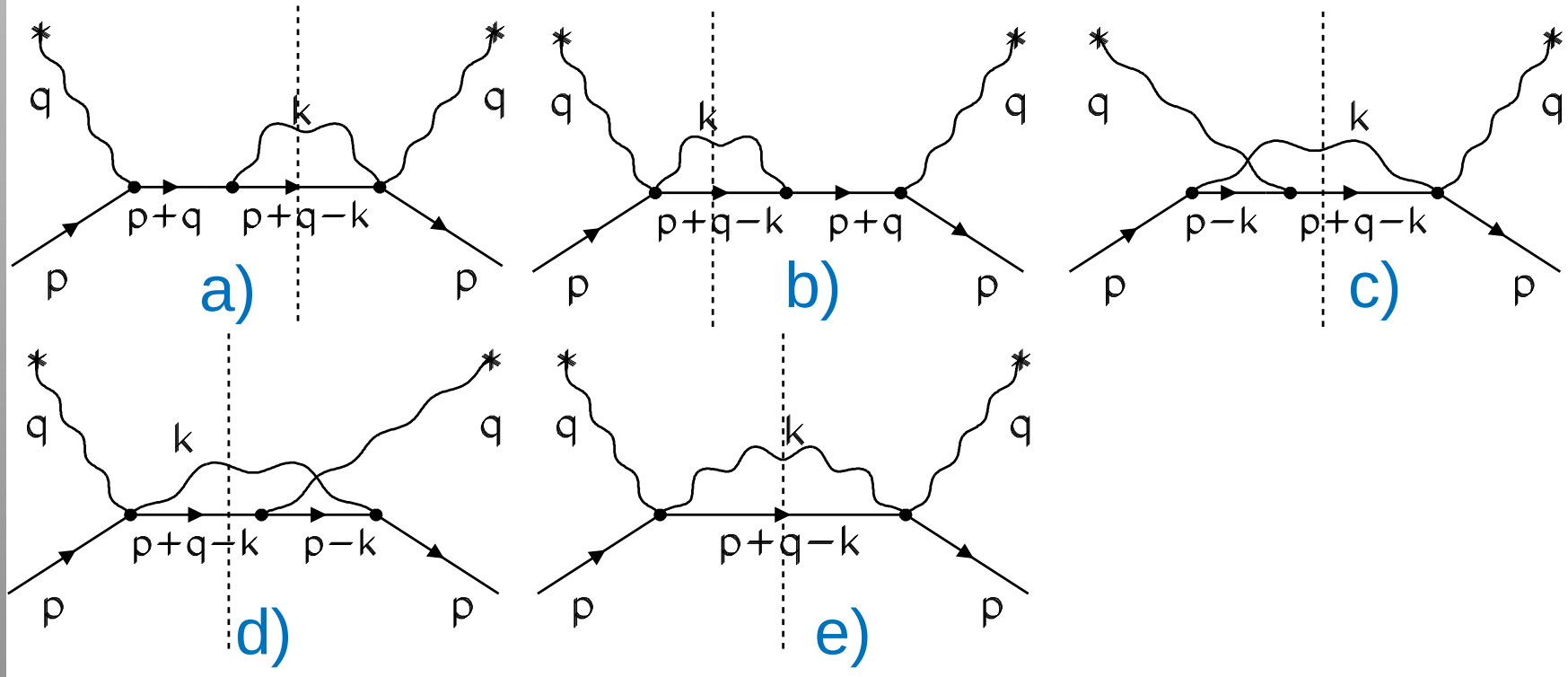
$$\Gamma^{pol}_{\sigma\delta}(p_1, q_1, p_2, q_2) = i(\tilde{\alpha}_{\sigma\delta}(p_1, q_1, p_2, q_2) + \tilde{\alpha}_{\delta\sigma}(p_1, -q_2, p_2, -q_1)).$$

This vertex obeys the following convolution:

$$q_2^{\sigma}\Gamma^{pol}_{\sigma\delta}(p_1, q_1, p_2, q_2) = 0, \; q_1^{\delta}\Gamma^{pol}_{\sigma\delta}(p_1, q_1, p_2, q_2) = 0,$$

e.i. the current conservation low is performed.

Feynman graphs for the lowest order VVCS amplitudes



The imaginary parts of these diagrams contribute to the hadronic tensor for inclusive DIS. It should be noticed that contribution from diagram (e) is negligible ($\sim 10^{-8}$ fm^6) and can be dropped.

Contribution to VVCS amplitudes

Performing cut over dash lines in diagrams (a-d) the imaginary part of amplitudes can be presented as

$$\mathrm{Im}\, T = \pi^2 \varepsilon^*_\nu(q)\varepsilon_\mu(q) \int d\Theta \bar{u}(p) \left[\frac{\Gamma^{\mu\nu\ a}_{VVCS} + \Gamma^{\mu\nu\ b}_{VVCS}}{(p+q)^2 - M^2} + \frac{\Gamma^{\mu\nu\ c}_{VVCS} + \Gamma^{\mu\nu\ d}_{VVCS}}{(p-k)^2 - M^2} \right] u(p) = \varepsilon^*_\nu(q)\varepsilon_\mu(q) T^{IVVCS}_{\mu\nu},$$

where the contribution from diagram (a)

$$\Gamma^{\mu\nu\ a}_{VVCS} = \Gamma^{pol\ \nu\alpha}(p+q-k,k,p,q)(\hat{p}+\hat{q}-\hat{k}+M)\Gamma^{el}_{\alpha}(-k) \times (\hat{p}+\hat{q}+M)\Gamma^{el\ \mu}(q),$$

$\Gamma^{el}_{\mu}(q) = -ie\left(F_D(-q^2)\gamma_\mu + F_P(-q^2) i\sigma_{\mu\alpha}q^\alpha/2M\right)$ is the usual elastic vertex. The contribution from other diagrams can be found by the same way.

The phase space

$$d\Theta = \frac{d^4k}{(2\pi)^4}\delta(k^2)\delta((p+q-k)^2 - M^2) = \frac{(S_x - Q^2)d\tau}{32\pi^3\tau^2\sqrt{\lambda_q}},$$

where we integrate over azimuthal photonic angle ϕ_k.
The invariants have a form:

$$Q^2 = -q^2,\ S_x = 2p\cdot q,\ \lambda_q = S_x^2 + 4M^2Q^2,\ \tau = \frac{k\cdot(p+q)}{k\cdot p}.$$

The limits of integration over τ read:

$$\tau_{max/min} = 1 + \frac{S_x \pm \sqrt{\lambda_q}}{2M^2}.$$

Contribution to hadronic tensor in DIS

The contribution of the presented above amplitude to the hadronic tensor can be calculated in a following way:

$$W_{\mu\nu}^{VVCS} = \frac{2}{M}\mathrm{Tr}[T_{\mu\nu}^{IVVCS}] = \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\mathrm{Im}\ T_1$$

$$+\frac{1}{p\cdot q}\left(p_\mu - \frac{p\cdot q}{q^2}q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2}q_\nu\right)\mathrm{Im}\ T_2$$

$$+\frac{i}{M}\varepsilon^{\mu\nu\alpha\beta}q_\alpha\eta_\beta\ \mathrm{Im}\ S_1 + \frac{i}{M}\varepsilon^{\mu\nu\alpha\beta}q_\alpha(p\cdot q\ \eta_\beta - \eta\cdot q p_\beta)\mathrm{Im}\ S_2.$$

Contribution to hadronic tensor in DIS

Using the standard projection operator technics as well as relations:

$$\mathrm{Im}\ T_1 = \frac{K}{4\pi}\sigma_T,\ \mathrm{Im}\ T_2 = \frac{\nu}{M}\frac{Q^2}{\nu^2+Q^2}\frac{K}{4\pi}(\sigma_T+\sigma_L)$$

$$\mathrm{Im}\ S_1 = \frac{\nu M}{\nu^2+Q^2}\frac{K}{4\pi}\left(\sigma_{TT}+\frac{Q}{\nu}\sigma_{LT}\right)$$

$$\mathrm{Im}\ S_2 = -\frac{M^2}{\nu^2+Q^2}\frac{K}{4\pi}\left(\sigma_{TT}-\frac{\nu}{Q}\sigma_{LT}\right)$$

we can extract $K\sigma_T$, $K\sigma_L$, $K\sigma_{TT}$ and $K\sigma_{LT}$.

Contribution to hadronic tensor in DIS

Taking into account the asymptotic limit

$$\lim_{\nu \gg Q,M} K\sigma_T = K\sigma_T^{\nu} \neq 0, \quad \lim_{\nu \gg Q,M} K\sigma_L = 0,$$

$$\lim_{\nu \gg Q,M} K\sigma_{TT} = K\sigma_{TT}^{\nu} \neq 0, \quad \lim_{\nu \gg Q,M} K\sigma_{LT} = K\sigma_{LT}^{\nu} \neq 0$$

we can define the next absorption cross sections that trend to zero for $\nu \to \infty$: $K\hat{\sigma}_T = K(\sigma_T - \sigma_T^{\nu})$, $K\hat{\sigma}_{TT} = K(\sigma_{TT} - \sigma_{TT}^{\nu})$ and $K\hat{\sigma}_{LT} = K(\sigma_{LT} - \sigma_{LT}^{\nu})$ to calculate Q^2-dependencies of forward polarizabilities.

Q^2-dependences of polarizabilities for forward VVCS scattering

$\alpha(Q^2)+\beta(Q^2)=\frac{1}{2\pi^2}\int\limits_{\nu_0}^{\infty} K\hat{\sigma}_T\frac{d\nu}{\nu^3}$, for sum of electric and magnetic polarizabilities;

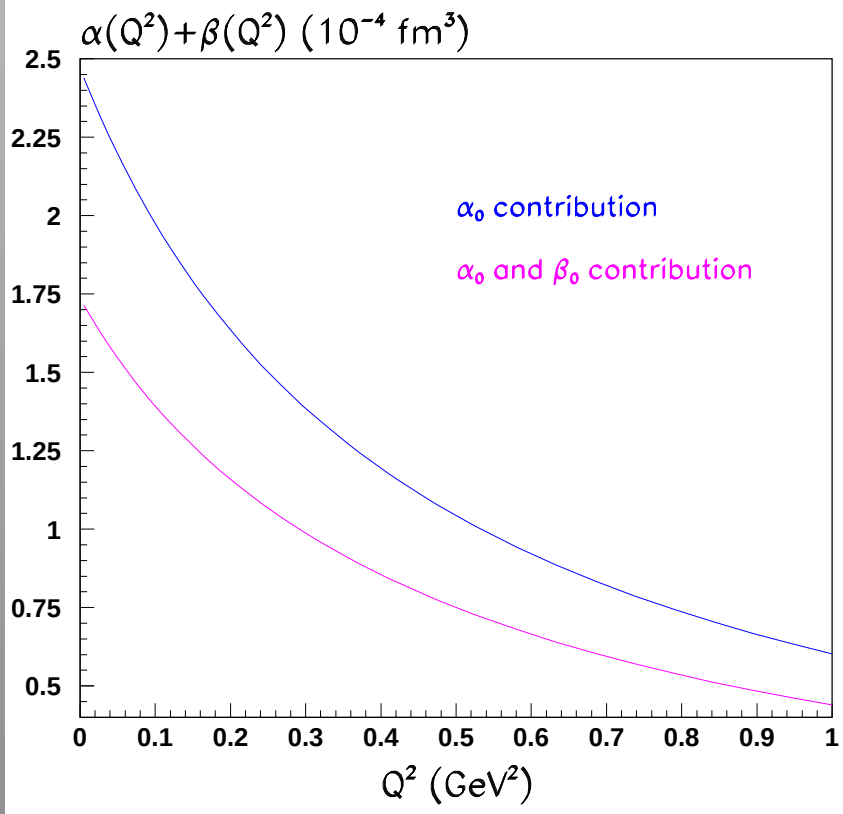
$\alpha_L(Q^2)=\frac{1}{2\pi^2}\int\limits_{\nu_0}^{\infty} K\sigma_L\frac{d\nu}{\nu^3}$, for longitudinal polarizability;

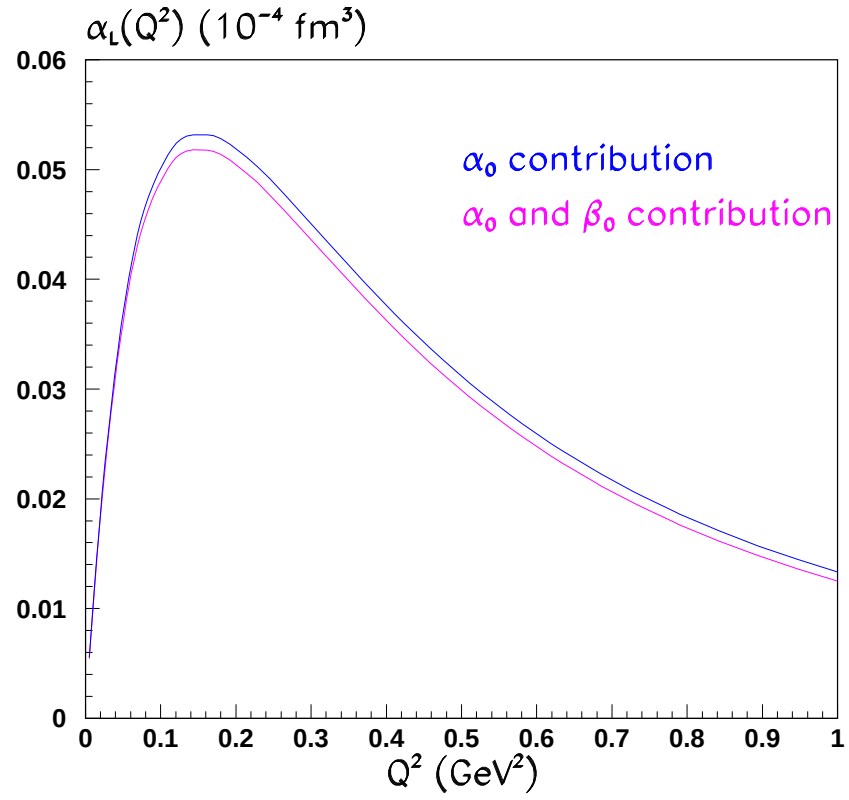
$\gamma_0(Q^2)=\frac{1}{2\pi^2}\int\limits_{\nu_0}^{\infty} K\hat{\sigma}_{TT}\frac{d\nu}{\nu^4}$, for spin polarizability;

$\delta_{LT}(Q^2)=\frac{1}{2\pi^2}\int\limits_{\nu_0}^{\infty} K\hat{\sigma}_{LT}\frac{d\nu}{\nu^3 Q}$, for longitudinal-transverse polarizability.

Here $\nu_0=m_\pi+(m_\pi^2+Q^2)/2M$ is pion threshold for DIS.

Q^2-dependences of polarizabilities for forward VVCS scattering

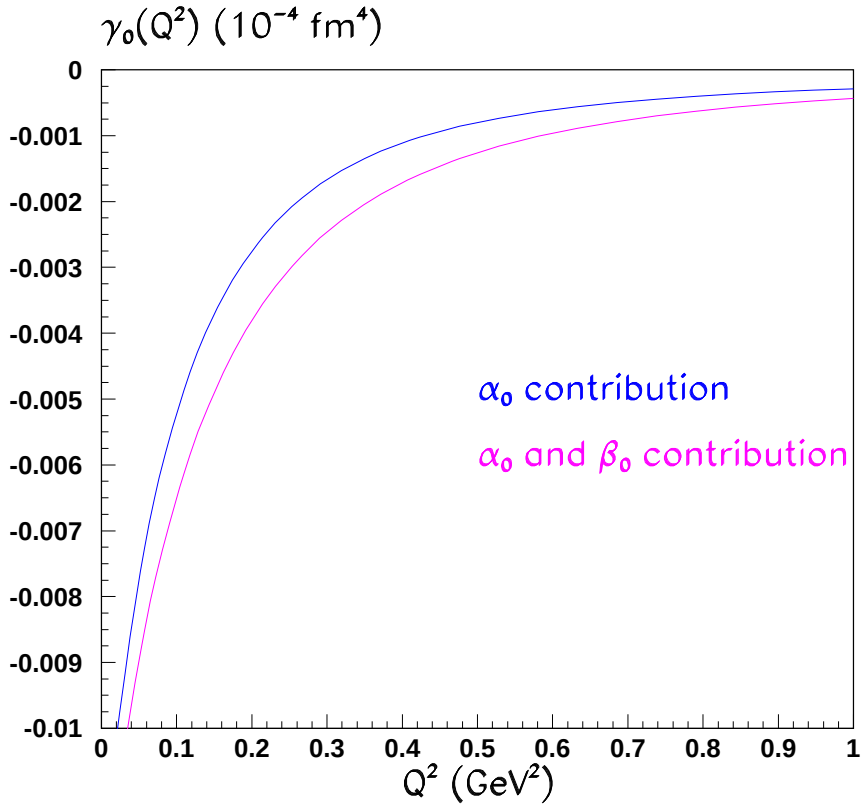


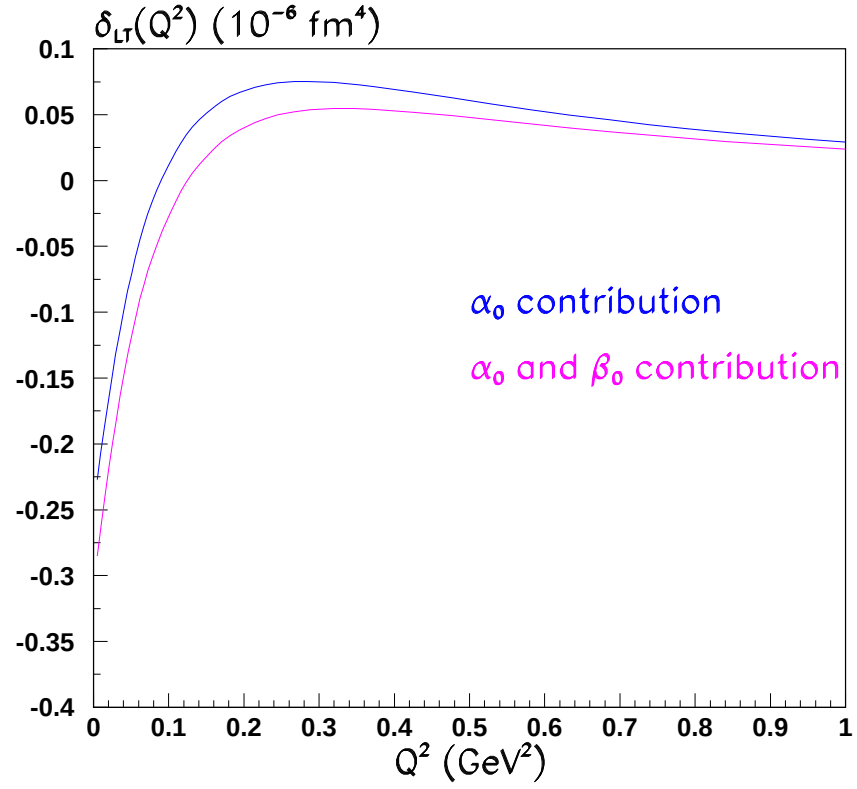


Q^2 dependence of the polarizabilities $\alpha + \beta$ and α_L for the proton.

Radiative correction to Baldin's sum rule 12%!!!

Q^2-dependences of polarizabilities for forward VVCS scattering





Q^2 dependence of the polarizabilities γ_0 and δ_{LT} for the proton

Conclusions

- On the basis of the relativistic electrodynamics of continuous media formalism and main relativistic quantum field theory principles the covariant Lagrangian of electromagnetic field interaction with polarizable 1/2-spin particles have been obtained. This Lagrangian let us to determine canonical and metric energy-momentum tensors as well as low-energy Compton scattering amplitude.
- The applications of this Lagrangian for calculations of static polarizability contribution to the imaginary part of doubly virtual Compton scattering amplitude off the proton have been demonstrated.

Conclusions

Numerical analysis shows that:

- Radiative correction to Baldin's sum rule 12%!!!
- The electric α_0 and magnetic β_0 static polarizabilities contribute not only to $\alpha(Q^2)+\beta(Q^2)$ and $\alpha_L(Q^2)$ but to $\gamma_0(Q^2)$ and $\delta_{LT}(Q^2)$ as well;
- The behavior of α_0 and β_0 contribution to $\alpha(Q^2)+\beta(Q^2)$ and $\alpha_L(Q^2)$ agrees with the results obtained in other channels and presented by [D. Drechsel, B. Pasquini, M. Vanderhaeghen];

Conclusions

- The behavior of α_0 and β_0 contribution to $\gamma_0(Q^2)$ and $\delta_{LT}(Q^2)$ disagrees only at low Q^2 with the results obtained in other channels and presented by [D. Drechsel, B. Pasquini, M. Vanderhaeghen];
- Similar to [D. Drechsel, B. Pasquini, M. Vanderhaeghen] in real photon limit $Q^2 \to 0$ we found that $\alpha_L(0) = 0$ while $\delta_{LT}(0) \neq 0$;
- It will be interesting to compare our result with experimental data (f.e. CLAS detector at JLab);

Conclusions

- In order to estimate the static polarizabilities contribution to VVCS correctly it is necessary to consider $\gamma_{E1,E2,M1,M2}$ contribution too. Now basing on the corresponding principle between the moving medium electrodynamic and quantum field theory the Lagrangian with this interaction is under construction.

Acknowledgments

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