Some puzzles in Low Energy Processes

A.E. Dorokhov (JINR)

Introduction

Current status of muon (g-2)

Rare $\pi^0 \to e^+e^-$ decay; Theory confronts KTeV data

Other $P \to l^+l^-$ decays

3σ Epidemy: Scalar FF in $K_{\mu3}$, Vus from τ decays, B-factory physics

Conclusions

Introduction

Abnormal people are looking for traces of Extraterrestrial Guests
Abnormal Educated people are looking for hints of New Physics

Cosmology tell us that 95% of matter is not described in text-books yet

Two search strategies:
1) High energy physics to excite heavy degrees of freedom.
No any evidence till now. Waiting for LHC era.
2) Low energy physics to produce Rare processes in view of huge statistics.
There are some rough edges of SM.

Generation of Mass!

I will discuss few of them shortly:
$\pi 0 \rightarrow e+e-$ (fresh)
$(g\text{-}2)_\mu$
Vus from τ decays

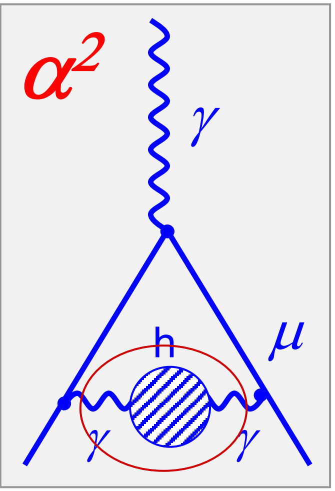
Anomalous magnetic moment of muon

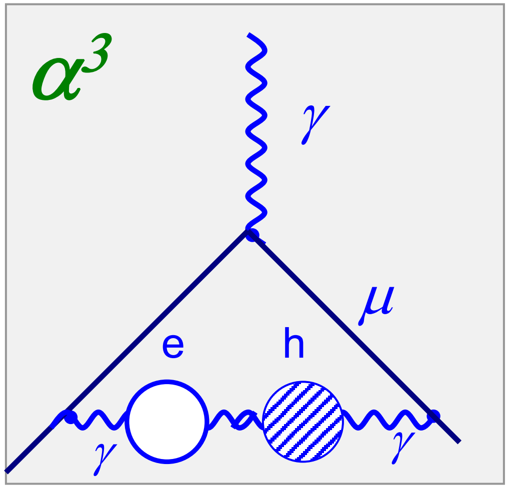
From BNL E821 experiment (1999-2006)

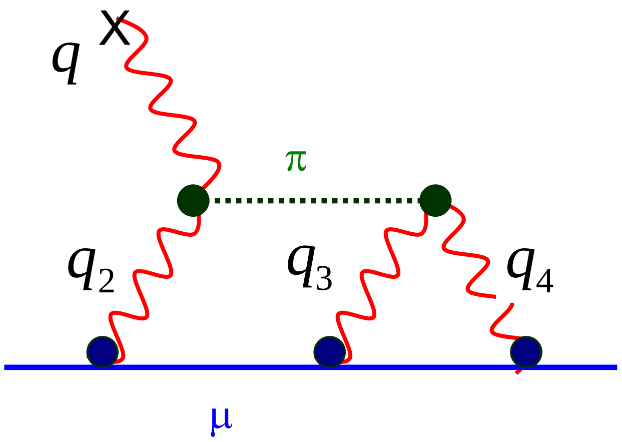
$$a_\mu^{BNL} = 11\ 659\ 208.0(5.4)(3.3) \bullet 10^{-10}$$

Standard Model $a_\mu^{SM} = a_\mu^{QED} + a_\mu^{EW} + a_\mu^{Hadr} + \ldots = 11\ 659\ 178.5(63) \bullet 10^{-10}$

predicts the result which is 3.4σ below the experiment

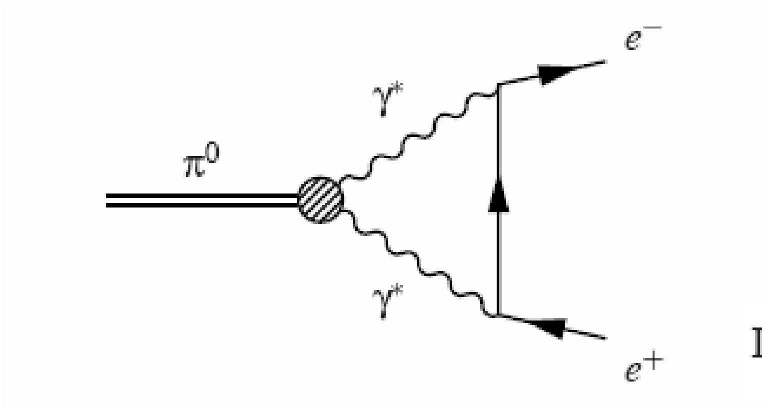






Rare Pion Decay $\pi^0 \rightarrow e+e--$ *from KTeV*

PRD (2007)



Lowest order diagram

One of the simplest process for theory

From KTeV E799-II experiment at Fermilab experiment (1997-2007)

$$B^{\mathrm{KTeV}}_{\pi \rightarrow e^+e^-} = (7.49 \pm 0.29 \pm 0.25) \cdot 10^{-8}$$

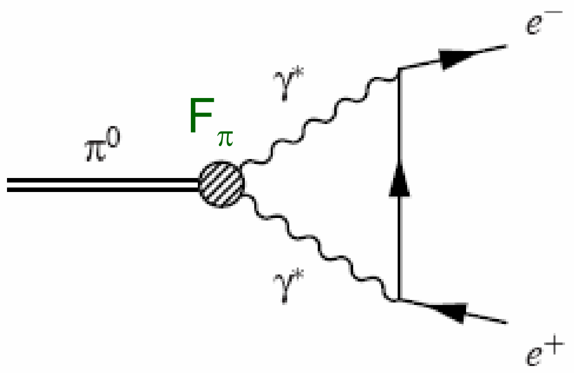
99-00' set,

The result is based on observation of 794 candidate $\pi_0 \rightarrow$ e+e- events using $K_L \rightarrow 3\pi_0$ as a source of tagged π_0s.
The older data used 275 events with the result:

$$B^{\mathrm{KTeV}}_{\pi \rightarrow e^+e^-}(\mathrm{old}) = (7.04 \pm 0.46 \pm 0.28) \cdot 10^{-8}$$

97' set

Classical theory of $\pi^0 \to e^+e^-$ decay



Drell (59'), Berman,Geffen (60'),
Quigg,Jackson (68')

Bergstrom,et.al. (82') dispersion approach
Savage, Luke, Wise (92') χPT

$$R\left(\pi^0 \to e^+e^-\right) = \frac{B\left(\pi^0 \to e^+e^-\right)}{B\left(\pi^0 \to \gamma\gamma\right)} = 2\beta\left(m_\pi^2\right)\left(\frac{\alpha}{\pi}\frac{m_e}{m_\pi}\right)^2\left[\left(\mathrm{Re}\,\mathcal{A}\left(m_\pi^2\right)\right)^2 + \left(\mathrm{Im}\,\mathcal{A}\left(m_\pi^2\right)\right)^2\right]$$

$$\beta\left(q^2\right) = \sqrt{1 - 4\frac{m_e^2}{q^2}}$$

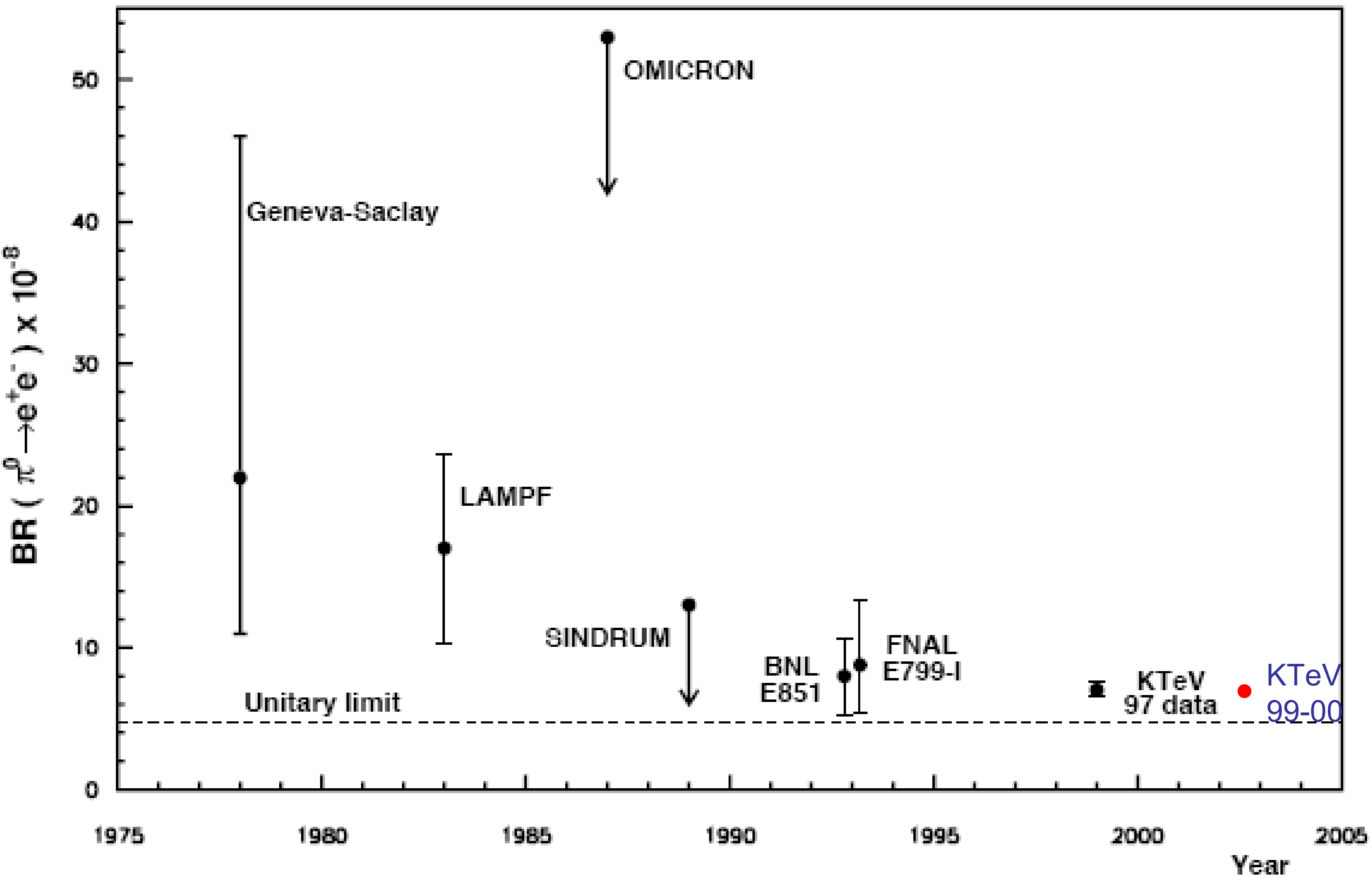
$$\mathcal{A}\left(q^2\right) = \frac{2i}{q^2}\int \frac{d^4k}{\pi^2}\frac{q^2k^2 - (qk)^2}{\left(k^2 + i\varepsilon\right)\left((k-q)^2 + i\varepsilon\right)\left((k-p)^2 - m_e^2 + i\varepsilon\right)} F_\pi\left(k^2, (k-q)^2\right)$$

$$\mathrm{Im}\,\mathcal{A}\left(q^2\right) = \frac{\pi}{2\beta\left(q^2\right)}\ln\left(\frac{1-\beta\left(q^2\right)}{1+\beta\left(q^2\right)}\right)$$

The Imaginary part is Model Independent

From condition $|\mathcal{A}|^2 \geq (\mathrm{Im}\,\mathcal{A})^2$ **one has the *unitary limit***

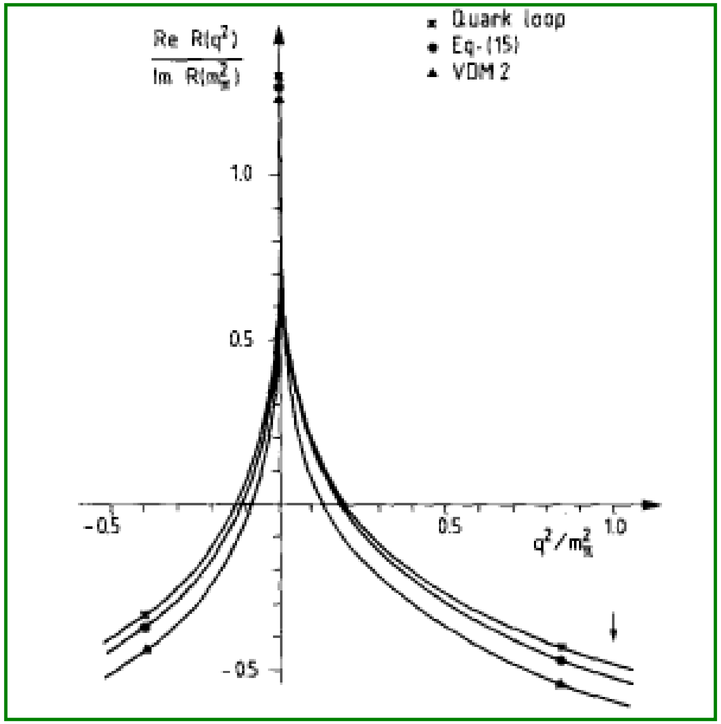
$$R\left(\pi^0 \to e^+e^-\right) \geq R^{\mathrm{unitary}}\left(\pi^0 \to e^+e^-\right) = 4.75 \cdot 10^{-8}.$$



Dispersion approach (Bergstrom et.al.(82))

$$\mathrm{Re}\,\mathcal{A}\left(q^2\right) = \mathrm{Re}\,\mathcal{A}\left(q^2=0\right) + \frac{q^2}{\pi}\int_0^\infty ds \frac{\mathrm{Im}\,\mathcal{A}\left(s\right)}{s\left(s-q^2\right)}.$$

$$\mathrm{Re}\,\mathcal{A}\left(q^2\right) = \mathrm{Re}\,\mathcal{A}\left(q^2=0\right) + \frac{1}{\beta_e\left(q^2\right)}\left[\frac{1}{4}\ln^2\left(y\right) + \frac{\pi^2}{12} + \mathrm{Li}_2\left(-y\right)\right], \qquad y = \frac{1-\beta_e\left(q^2\right)}{1+\beta_e\left(q^2\right)}.$$



The Real Part is known up to Constant

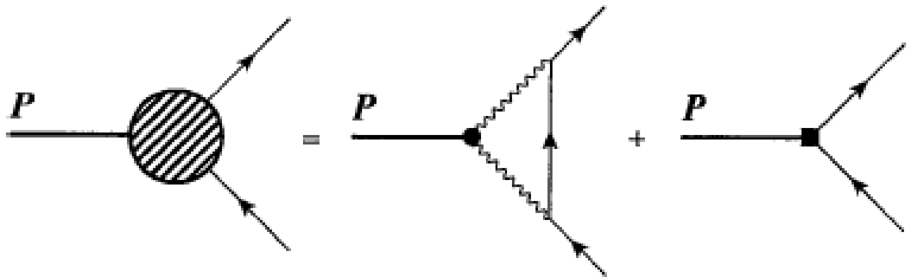
The Constant is the Amplitude in Soft Limit $q^2 \to 0$

In general it is determined in Model Dependent way

$$\mathrm{Re}\,\mathcal{A}\left(m_\pi^2\right) = \mathrm{Re}\,\mathcal{A}\left(q^2=0\right) + \ln^2\left(\frac{m_e}{m_\pi}\right) + \frac{\pi^2}{12} \approx \mathrm{Re}\,\mathcal{A}\left(q^2=0\right) + 31.9$$

χPT approach (D'Ambrosio,Espriu(86),Savage,Luke,Wise(92))

$$\mathcal{L}_{P\ell^+\ell^-} = \frac{3i}{32}\left(\frac{\alpha}{\pi}\right)^2 \bar{\ell}\gamma^\mu\gamma_5\ell \times [\chi_1 \mathrm{tr}(Q_R Q_R D_\mu U U^\dagger - Q_L Q_L D_\mu U^\dagger U) + \chi_2 \mathrm{tr}(U^\dagger Q_R D_\mu U Q_L - U Q_L D_\mu U^\dagger Q_R)].$$



$$\mathrm{Re}[F(P \to l^+ l^-)] = \frac{1}{4\beta}\ln^2\left(\frac{1-\beta}{1+\beta}\right) + \frac{1}{\beta}\mathrm{Li}_2\left(\frac{\beta-1}{\beta+1}\right) + \frac{\pi^2}{12\beta} + 3\ln\left(\frac{m_l}{\mu}\right) + \chi(\mu)$$

$\chi(\mu) = -[\chi_1^r(\mu) + \chi_2^r(\mu) + 14]/4$

is unknown LE constant

I. The Decay Amplitude in Soft limit $q^2 \to 0$

A.D., M.A.Ivanov
PRD 07'

$$\mathrm{Re}\,\mathcal{A}\left(q^2=0\right) = 3\ln\left(\frac{m_e}{\mu}\right) + \chi_P\left(\mu\right)$$

$$\chi_P\left(\mu\right) = -\frac{5}{4} + \frac{3}{2}\int_0^\infty dt \ln\left(\frac{t}{\mu^2}\right)\frac{\partial F_\pi\left(t,t\right)}{\partial t} = -\frac{5}{4} - \frac{3}{2}\left[\int_0^{\mu^2} dt \frac{F_\pi\left(t,t\right)-1}{t} + \int_{\mu^2}^\infty dt \frac{F_\pi\left(t,t\right)}{t}\right]$$

The unknown constant is expressed as inverse moment of Pion Transition FF!!!

The accuracy is of order $O(m_e/m_\rho)^2$.
Thus the amplitude is fully reconstructed!

$$\mathcal{A}\left(q^2\right) = \frac{2i}{q^2}\int \frac{d^4k}{\pi^2}\frac{q^2k^2-\left(qk\right)^2}{\left(k^2+i\varepsilon\right)\left(\left(k-q\right)^2+i\varepsilon\right)\left(\left(k-p\right)^2-m_e^2+i\varepsilon\right)}F_\pi\left(k^2,\left(k-q\right)^2\right)$$

$$m_e \ll m_\pi \ll \Lambda \sim m_\rho$$

II. CLEO data and Lower Bound on Branching

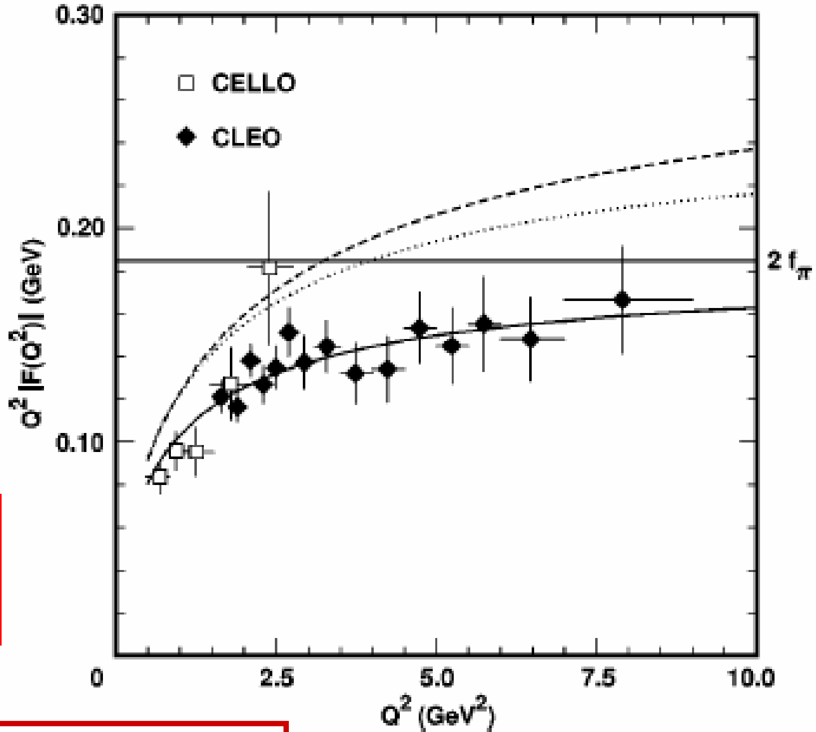
Use inequality $F_\pi(t,t) < F_\pi(t,0)$ at spacelike $t > 0$

and CLEO data (98')

$$F_\pi^{\text{CLEO}}(t,0) = \frac{1}{1 + t/s_0^{\text{CLEO}}},$$

$$s_0^{\text{CLEO}} = (776 \pm 22 \quad \text{MeV})^2$$

$$\text{Re}\,\mathcal{A}(q^2 = 0) > -\frac{3}{2}\ln\left(\frac{s_0^{\text{CLEO}}}{m_e^2}\right) - \frac{5}{4} = -23.2 \pm 0.1$$



$$R\left(\pi^0 \to e^+e^-\right) \geq R^{\text{CLEO}}\left(\pi^0 \to e^+e^-\right) = (5.91 \pm 0.02) \cdot 10^{-8}$$

$$R^{\text{KTeV}} = (7.58 \pm 0.40) \cdot 10^{-8}$$

$$R\left(\pi^0 \to e^+e^-\right) \geq R^{\text{unitary}}\left(\pi^0 \to e^+e^-\right) = 4.75 \cdot 10^{-8}.$$

III. $F_\pi(t,t)$ general arguments

Assume $$F_\pi(t,t) = \frac{1}{1+t/s_1}$$ **then** $$\mathrm{Re}\,\mathcal{A}^{\mathrm{theory}}(q^2=0) = -\frac{3}{2}\ln\left(\frac{s_1}{m_e^2}\right) - \frac{5}{4}$$

1. From $$-\frac{\partial F_\pi(t,t)}{\partial t}\bigg|_{t=0} = -2\frac{\partial F_\pi(t,0)}{\partial t}\bigg|_{t=0}$$ **one has** $$s_1 = s_0/2$$

2. From OPE QCD

$$F_\pi^{\mathrm{OPE}}(t,0)\big|_{t\to\infty} = 8\pi^2 f_\pi^2 \frac{1}{t},$$
$$F_\pi^{\mathrm{OPE}}(t,t)\big|_{t\to\infty} = \frac{8\pi^2 f_\pi^2}{3}\frac{1}{t}$$

one has $$s_1^{OPE} = s_0^{OPE}/3$$

F(t,0) → F(t,t) reduces to rescaling

It follows $$\mathrm{Re}\,A^{\mathrm{theory}}(q^2=0) = -21.9 \pm 0.3$$

$$B^{\mathrm{theory}}(\pi^0 \to e^+e^-) = (6.2 \pm 0.1)\bullet 10^{-8}$$

3.2σ below data!!

$$B^{\mathrm{KTeV}} = (7.49 \pm 0.39)\bullet 10^{-8}$$

It would required change of s_0 scale by factor more then 10!

A. Fπ(t,t) QCD sum rules (V.Nesterenko, A.Radyushkin, YaF 83')

$$F_{\pi}^{\mathrm{QCDsr}}(t,t) = 2\int_0^{s_0^{\mathrm{QCDsr}}} ds \int_0^1 dx \frac{x(1-x)t^2}{[x(1-x)s+t]^3} + \mathrm{v.c.},$$

From

$$-\frac{\partial F_{\pi}(t,t)}{\partial t}\bigg|_{t=0} = -2\frac{\partial F_{\pi}(t,0)}{\partial t}\bigg|_{t=0}$$

and

$$\langle r^2\rangle_{\pi^0\gamma^*\gamma^*}^{\mathrm{QCDsr}} = -6\frac{\partial F_{\pi}^{\mathrm{QCDsr}}(t,t)}{\partial t}\bigg|_{t=0} = \frac{12}{s_0^{\mathrm{QCDsr}}}$$

one has

$$s_0^{\mathrm{QCDsr}} = s_0^{\mathrm{CLEO}}$$

$$\mathrm{Re}\, A^{\mathrm{QCDsr}}\left(q^2=0\right) = -\frac{3}{2}\ln\left(\frac{s_0^{\mathrm{CLEO}}}{\mathrm{m}_e^2}\right) + \frac{1}{4} = -21.7 \pm 0.1,$$

$$s_1^{\mathrm{QCDsr}} = \frac{s_0^{\mathrm{QCDsr}}}{e}$$

Nicely confirms general arguments!

B. $F\pi(t,t)$ VMD parametrization *(M.Knecht, A.Nyffeler, EPJC 01')*

$$F_{gVMD}(s,t) = \frac{4\pi^2 f_\pi^2}{3} \frac{(s+t)\,st - h_2 st + h_5 (s+t) + M_V^4 M_{V1}^4 h_7}{(M_V^2+s)(M_V^2+t)(M_{V1}^2+s)(M_{V1}^2+t)}$$

$$\langle r^2 \rangle_{\pi\gamma\gamma^*} = -6 \left.\frac{\partial F(t,0)}{\partial t}\right|_{t=0} = 6\left(\frac{1}{M_V^2} + \frac{1}{M_{V1}^2} - \frac{h_5}{M_V^2 M_{V1}^2}\frac{3}{4\pi^2 f_\pi^2}\right) = 0.39 \quad \mathrm{fm}^2$$

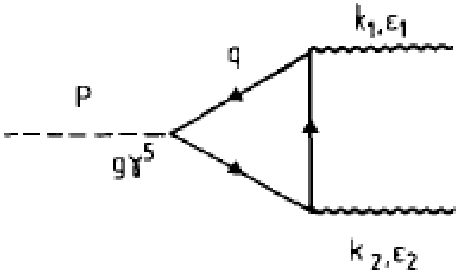
$$\mathrm{Re}\,\mathcal{A}_{g\mathrm{VMD}}(q^2=0) = -3\ln\left(\frac{M_V}{m_e}\right) + \frac{1}{4} + \frac{3r}{(r-1)^2} - \frac{3}{2}\frac{3r-1}{(r-1)^3}\ln r +$$

$$+\frac{4\pi^2 f_\pi^2}{M_V^2 (r-1)^3}\left(\frac{h_2}{2M_V^2}\left((r+1)\ln r - 2(r-1)\right) - \left(1 + \frac{h_5}{M_{V1}^2 M_V^2}\right)(r^2 - 1 - 2r\ln r)\right) = -21.94$$

$r = (M_{V1}/M_V)^2$.

Nicely confirms general arguments!

C. Fπ(t,t) Quark Models (Bergstrom 82')



$$F_\pi^{\text{quark}}(t,t) = \frac{2M_q^2}{\beta_q(t)\,t} \ln\left(\frac{\beta_q(t)+1}{\beta_q(t)-1}\right), \qquad \beta_q(t) = \sqrt{1+4\frac{M_q^2}{t}}$$

$$\mathrm{Re}\,\mathcal{A}_{\mathrm{QM}}(q^2=0) = 3\ln\left(\frac{m_e}{M_q}\right) - \frac{17}{4}$$

$$\mathrm{Re}\,\mathcal{A}_{\mathrm{QM}}(q^2=0) = -(23.4 \pm 0.5)$$ $$M_q = 300 \pm 50 \text{ MeV}$$

D. $F_\pi(t,t)$ Nonlocal Quark Models

$$\mathrm{Re}\,A^{\mathrm{NLQM}}\left(q^2=0\right)=-3\ln\left(\frac{2}{\mathrm{m_e}L}\right)-\frac{15}{4}-\frac{\pi^2}{12}-\frac{3}{2}B'(0),$$

$$B'(0)=1+4\int_0^\infty du\,\exp\left(-u^2\right)\ln(u)\left[u\cos(\xi u)+\frac{1}{2}\xi\sin(\xi u)\right],$$

$$L=3.12\ \mathrm{GeV}^{-1},\ \ \xi=\frac{2l}{L}=1.4$$

$$B^{\mathrm{NLQM}}\left(\pi^0\to e^+e^-\right)=5.38\bullet10^{-8}$$

(Efimov,Ivanov 81')

$$\mathrm{Re}\,A^{\mathrm{ILM}}\left(q^2=0\right)=-3\ln\left(\frac{M_q}{\mathrm{m_e}}\right)-\frac{5}{4}+\frac{3}{2}\gamma_E-\frac{3}{2}I\left(\rho M_q\right),$$

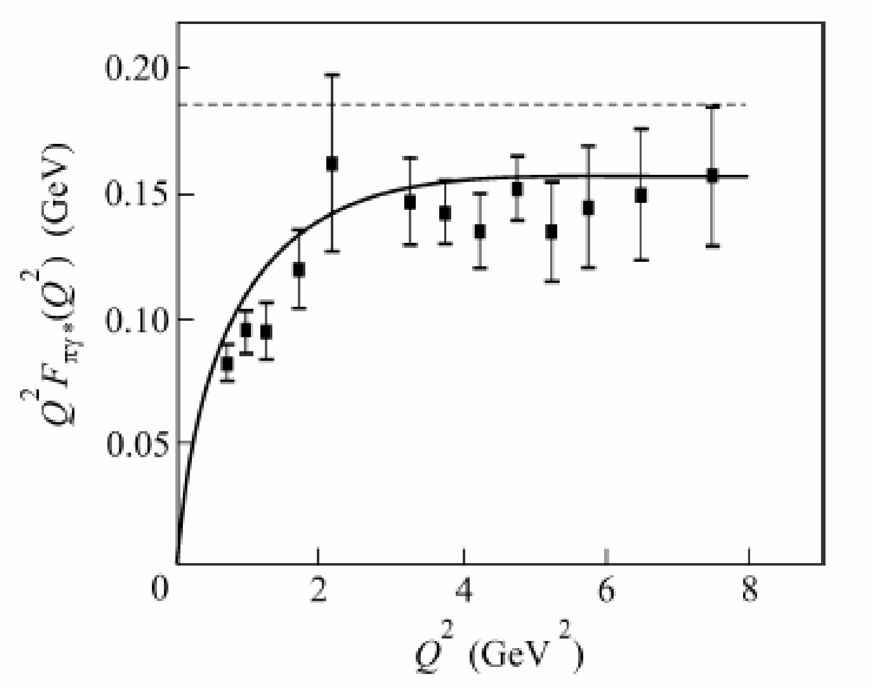
$$M_q=300\pm50\ \mathrm{MeV},\ \ \rho\approx1.7\ \mathrm{GeV}^{-1}$$

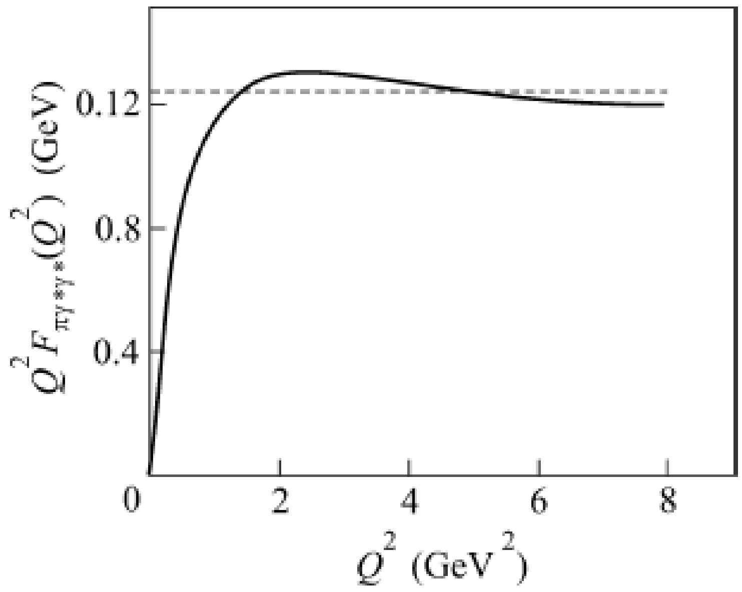
$$B^{\mathrm{ILM}}\left(\pi^0\to e^+e^-\right)=6.2\bullet10^{-8}$$

(A.Dorokhov 06')

Nicely confirms general arguments!

Instanton Model and Pion Transition FF





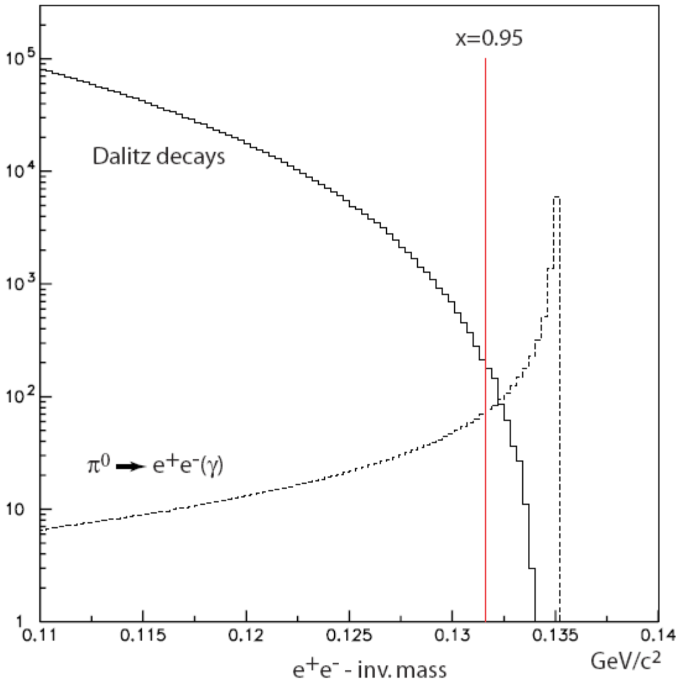
Theory vs KTeV experiment

	CLEO + OPE	QCDsr	gVMD	QM [12]	NχQM	NQM [10]	Experiment [1]
$-\mathcal{A}(q^2=0)$	21.9 ± 0.3	21.7 ± 0.1	21.9	23.4 ± 0.5	22.1 ± 0.5	24.5	18.6 ± 0.9
$B(\pi^0 \to e^+e^-) \times 10^8$	6.23 ± 0.09	6.21 ± 0.05	6.2	5.8 ± 0.2	6.1 ± 0.2	5.38	7.49 ± 0.38

$$B^{\text{theory}}\left(\pi^0 \to e^+e^-\right) = \left(6.2 \pm 0.1\right) \bullet 10^{-8}$$

$$B^{\text{KTeV}}\left(\pi \to e^+e^-\right) = \left(7.49 \pm 0.29 \pm 0.25\right) \bullet 10^{-8}$$

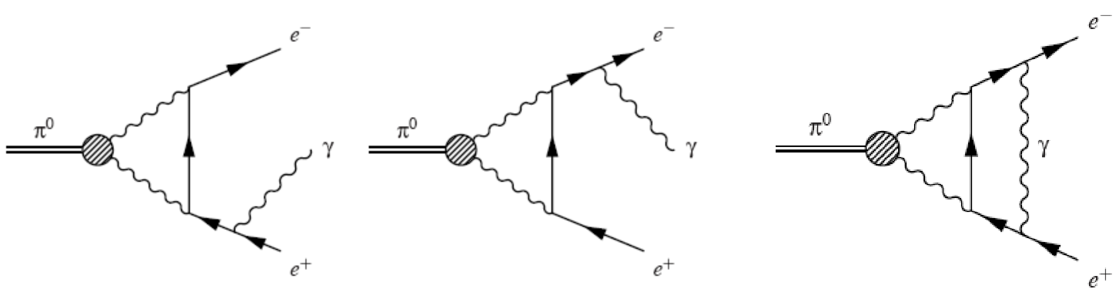
IV. Radiative Corrections *(Bergstrom 83' A.D.,Kuraev, Bystritsky, Secansky 07')*

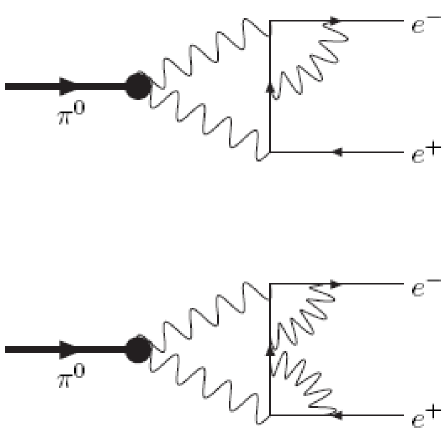


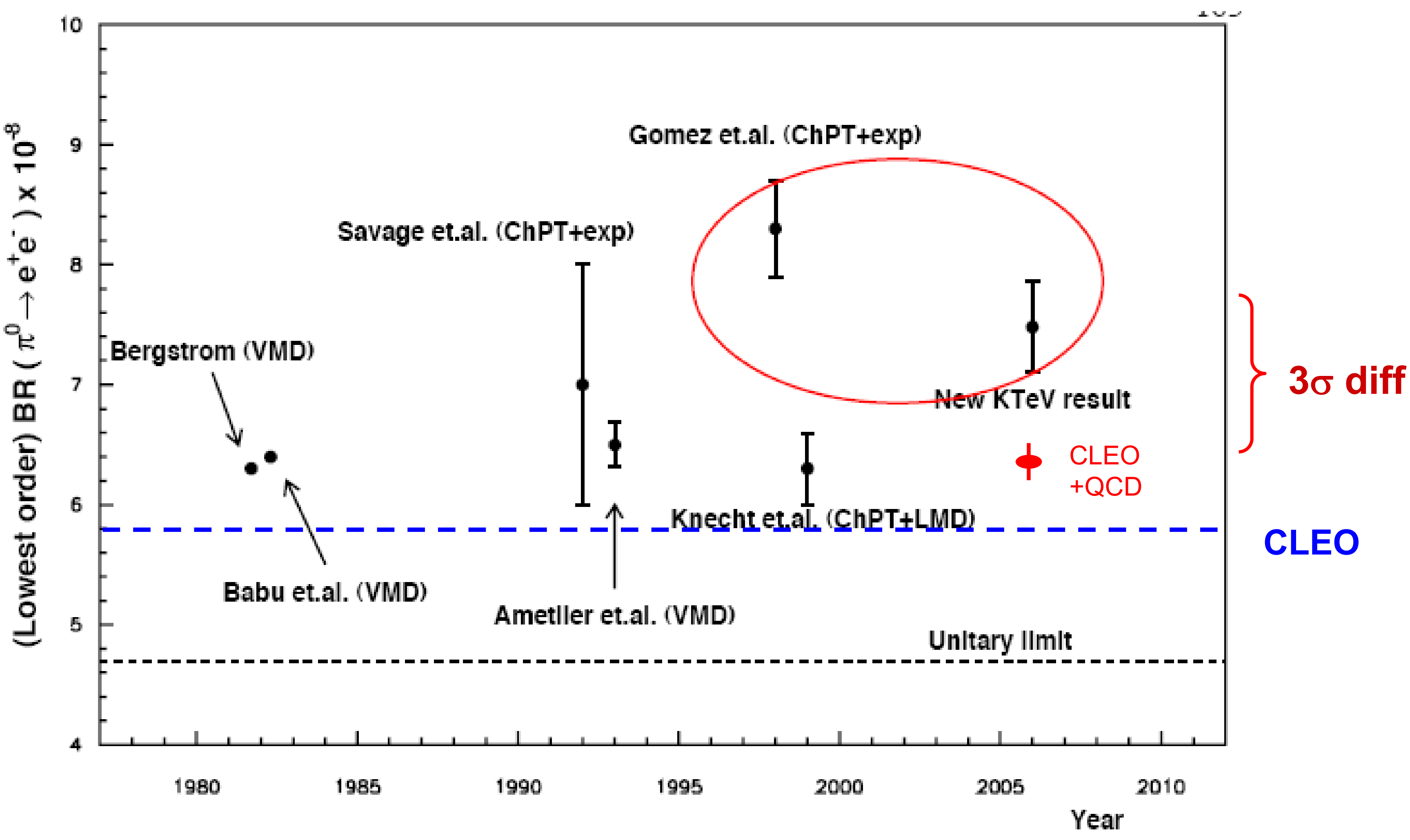
$$B(\pi^0 \to e^+e^-, x_D > 0.95) = (6.44 \pm 0.25 \pm 0.22) \times 10^{-8}$$

$$B^{\text{no-rad}}(\pi^0 \to e^+e^-) = (7.48 \pm 0.29 \pm 0.25) \times 10^{-8}$$

$$\frac{\Gamma^{\text{brem}} + \Gamma^{\text{virt}}}{\Gamma^0_{ee}} = \frac{\alpha}{\pi}\left\{\frac{3}{2}\ln\left(\frac{1-\beta_0}{1+\beta_0}\right) + \frac{9}{4}\right\} + O\left(\frac{m_e^2}{m_{\pi^0}^2}\right)$$ =-3.4%







What is next? It would be very desirable if PIBETA will confirm KTeV result
Also, Pion transition FF need to be more accurately measured.

Other P →l+l– decays

$$F_\eta^{\rm CLEO}\left(k^2, q^2=0\right) = \frac{1}{1+k^2/s_{0\eta}^{\rm CLEO}}, \qquad s_{0\eta}^{\rm CLEO} = (774 \pm 29 \quad \mathrm{MeV})^2,$$

TABLE II. Values of the branchings $B(P \to l^+ l^-)$ obtained in our approach and compared with the available experimental results.

B	Unitary bound	CLEO bound	CLEO + OPE	Experiment
$B(\pi^0 \to e^+ e^-) \times 10^8$	≥ 4.69	$\geq 5.85 \pm 0.03$	6.23 ± 0.09	7.49 ± 0.38 [1]
$B(\eta \to \mu^+ \mu^-) \times 10^6$	≥ 4.36	$\leq 6.23 \pm 0.12$	5.11 ± 0.20	5.8 ± 0.8 [7,32]
$B(\eta \to e^+ e^-) \times 10^9$	≥ 1.78	$\geq 4.33 \pm 0.02$	4.60 ± 0.06	...

Scalar form factor of $K_{\mu 3}$ ($K \to \pi\mu\nu$) decay from NA48 and KTeV and conflict with the low energy theorem by Callan and Treiman

$$f_0(M_K^2 - M_\pi^2) = \frac{F_K}{F_\pi} + O(m_u, m_d)$$

$$f_0\left(q^2 \to 0\right) = f_0\left(q^2 = 0\right)\left(1 + \lambda_0 \frac{q^2}{m_\pi^2} + O\left(q^4\right)\right)$$

Equivalent prediction for the FF curvature

$$\lambda_0^{\chi PT} = 0.0157(10),$$

but

$$\lambda_0^{NA48} = 0.0117(7)(1), \quad (2007)$$

$$\lambda_0^{kTeV} = 0.01372(131), \quad (2004)$$

conflict

$$\lambda_0^{Istra} = 0.0183(11)(6), \quad (2003)$$

$$\lambda_0^{KLOE} = 0.0156(26), \quad (2007)$$

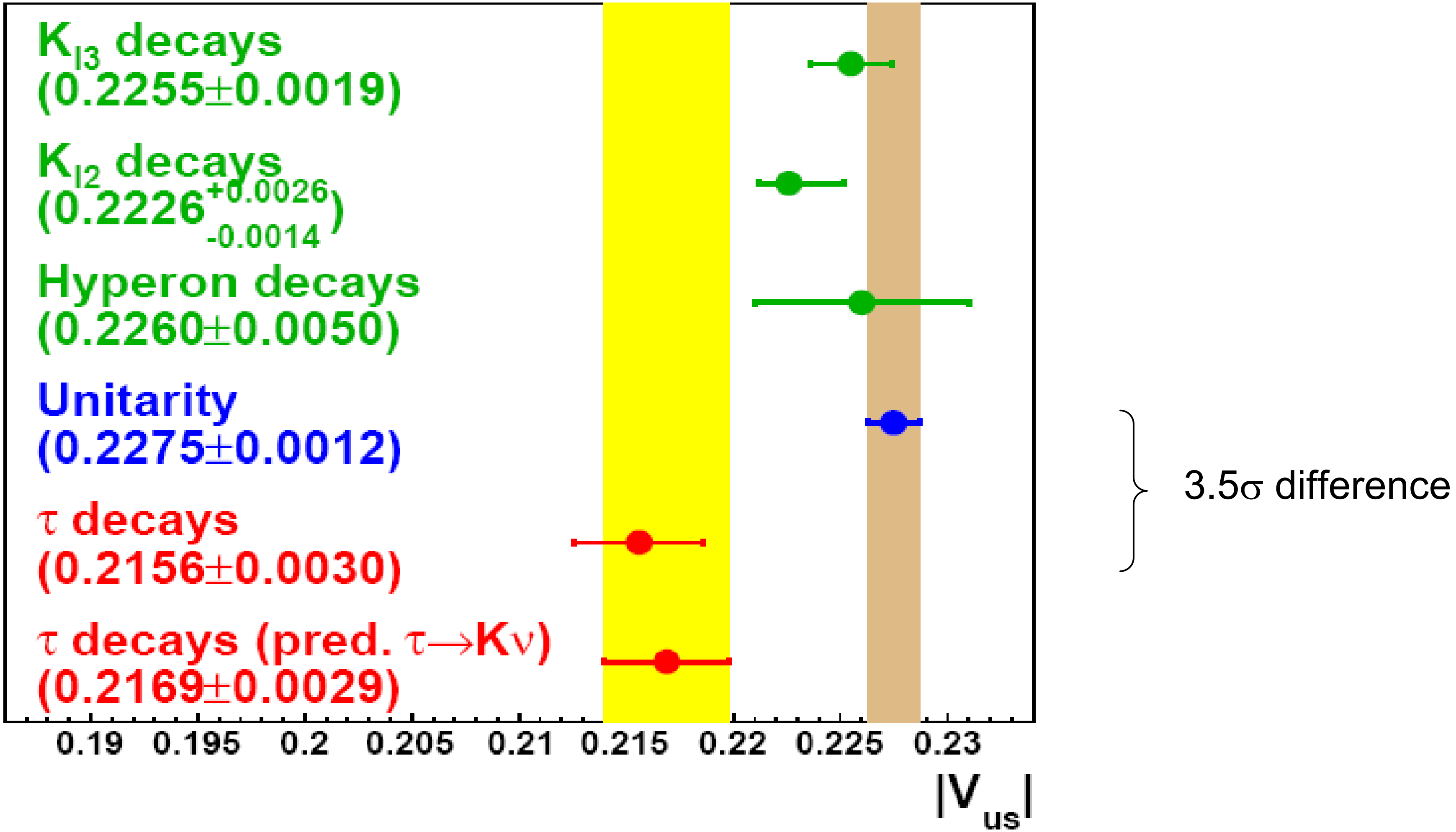
unpublished

$$f_0^{NA48}\left(m_K^2 - m_\pi^2\right) = 1.155(8)(13) \bullet f_+(0),$$

$$f_0^{\mathrm{Nucl}\beta\mathrm{Dec}}\left(m_K^2 - m_\pi^2\right) = 1.242(4) \bullet f_+(0)$$

Very strong isospin breaking?

Measurement of |*Vus*| using hadronic τ decays from *BABAR* & BELLE



Conclusions

The conclusion is that the experimental situation calls for clarification.
There are not many places where the Standard Model fails.
Hints at such failures deserve particular attention.

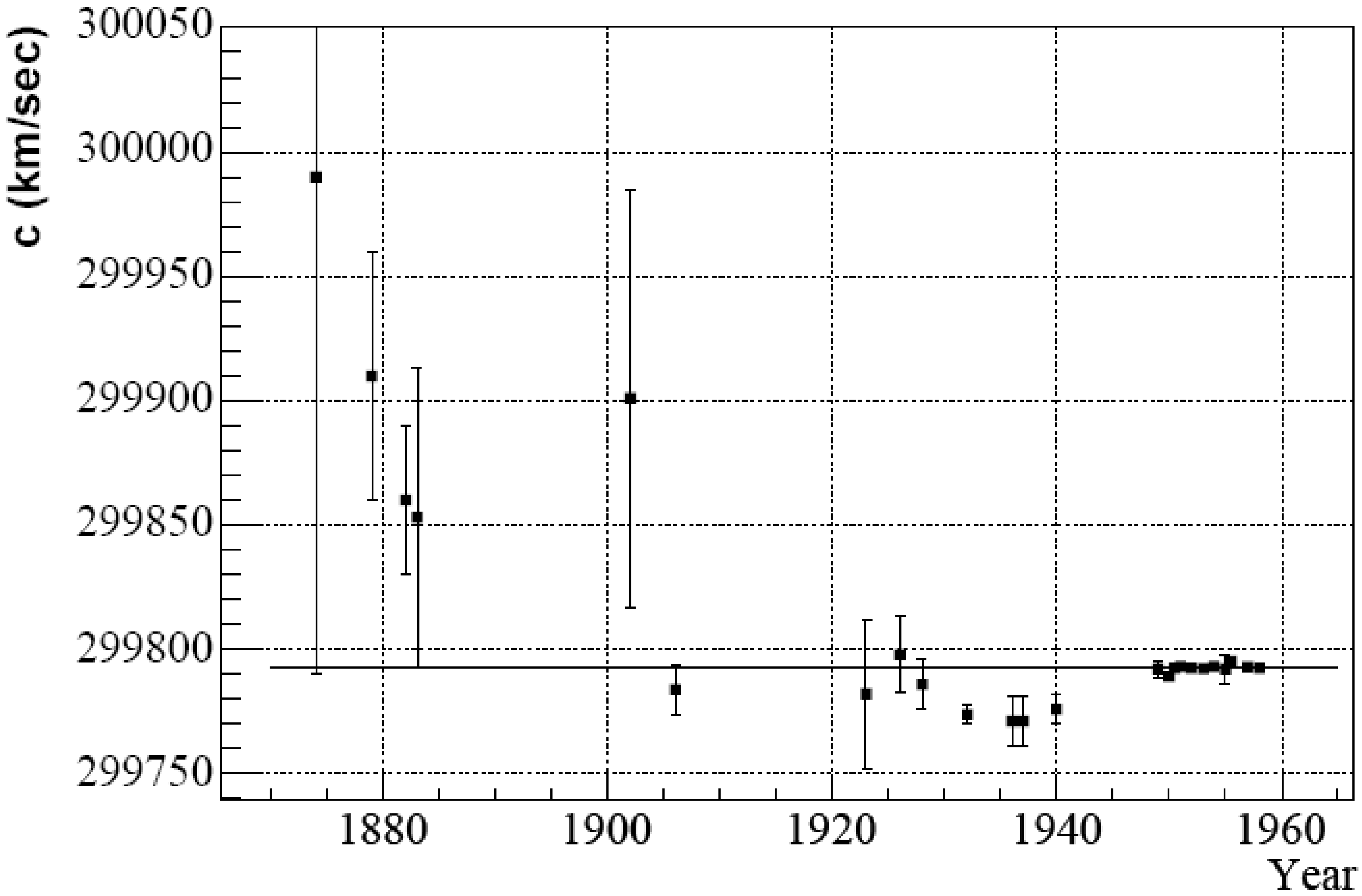
Perhaps new generation of high precision experiments and new accurate Calculations might help removing the dust.

Many B-factory results indicate interesting deviations from the SM.
One of the most compelling hints of new physics are the measurements of the time-dependent CP asymmetries in penguin dominated modes that turned out to be systematically smaller than the SM expectation.
The magnitude of the deviation ranges from 2.5σ to 4σ

New Physics
Still durty Strong Interaction
or
The measurements tend to cluster nearer the prior published averages than the 'final' value. (weather forecast style)

Much more experimental information is required to disentangle the various possibilities.

Summary of speed of light measurements



It is interesting that the series of four measurements from1930-1940 displays a 17km/sec systematic shift from the true value

Conclusions

The low-energy constant defining the dynamics of the process is expressed as the inverse moment of pion transition FF

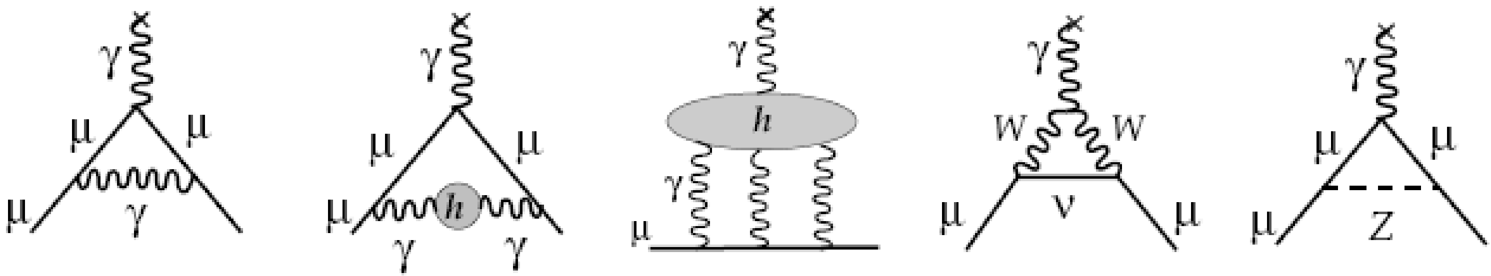
Data on pion transition form factor provide new bounds on decay branchings essentially improving the unitary ones.

QCD constraint further the change of scales in transition from asymmetric to symmetric kinematics of pion FF

We found 3σ difference between theory and KTeV data

If these results are confirmed, then the Standard Model
is in conflict with observation in one of those reactions which we thought
are best understood.

$$\Delta a_\mu^{(\mathrm{today})} = a_\mu^{(\mathrm{Exp})} - a_\mu^{(\mathrm{SM})} = (295 \pm 88) \times 10^{-11}$$



(a) (b) (c) (d) (e)

Effect	*Contribution* $\times 10^{11}$	*Future* σ
QED	$(116\,584\,718.09 \pm 0.14_{\mathrm{5loops}} \pm 0.08_{\alpha} \pm 0.04_{\mathrm{masses}}) \times 10^{-11}$	
Hadronic (lowest order)	$a_\mu^{(\mathrm{HVP;1})} = (6901 \pm 42_{\mathrm{exp}} \pm 19_{\mathrm{rad}} \pm 7_{\mathrm{QCD}})$	$\pm 30_{\mathrm{exp}} \pm 8_{\mathrm{rad}} \pm 7_{\mathrm{QCD}}$
Hadronic (higher order)	$a_\mu^{(\mathrm{HVP;h.o.})} = (-97.9 \pm 0.9_{\mathrm{exp}} \pm 0.3_{\mathrm{rad}})$	
Hadronic (light-by-light)	$a_\mu^{(\mathrm{HLLS})} = (110 \pm 40)$	16.5
Electroweak	$a_\mu^{(\mathrm{EW})} = (154 \pm 2_{\mathrm{M_H}} \pm 1_{\mathrm{had}})$	

Lightest observable SS particle mass