Observables for model-independent detections of Z' boson at the ILC

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ABSTRACT

The integral observables for model-independent detections of Abelian Z' gauge boson in $e^+e^- \to \mu^+\mu^-(\tau^+\tau^-)$ process with unpolarized beams at the ILC energies are proposed. They are based on the differential cross-section of deviations from the standard model predictions calculated with a low energy effective Lagrangian and taking into consideration the relations between the Z' couplings to the fermions. The cross-section exhibits angular distribution giving a possibility for introducing one- or two parameter observables which **effectively fit** the mass $m_{Z'}$, the axial-vector $a^2_{Z'}$ and the product of vector couplings $v_e v_\mu (v_e v_\tau)$. A discovery reach for the Z' is estimated for two of introduced observables. Determination of the basic Z' model is discussed.

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INTRODUCTION

Searching for new heavy particles beyond the energy scale of the standard model (SM) is one of the main goals of modern high energy physics. **Nowadays data of Tevatron and the LHC are analised. Discoveries of these experiments will be further investigated in details** at the ILC which will have energies of $\sim 500 - 1000$ GeV in the center-of-mass of beams.

One of expected heavy particles beyond the SM is Z' gauge boson which is related with an additional $\tilde{U}(1)$ group. It enters as a necessary element numerous GUT models like $SO(10)$, E_6 as well as superstrings, extra dimensions, etc.

Searches for this particle have been established already within the LEP data in either model-dependent or model-independent approaches, and the Tevatron data. Modern model-dependent measurements constrain that the mass $m_{Z'}$ to be larger than $2.5 - 2.9$ TeV [ATLAS, CMS].

So, at the ILC experiments the Z' will be investigated as a virtual state.

At present about hundred Z' models are discussed in the literature. In model-dependent searches established, only the most popular ones such as LR, ALR, χ, ψ, η, B - L, SSM, have been investigated and the particle mass estimated.

Most investigations devoted to model-dependent searches at the ILC deal with the polarized beams and corresponding observables are introduced.

As complementary way, a model-independent approach is very desirable. In this method not only the Z' mass but also the couplings to the SM fermions are unknown parameters which must be fitted in experiments. Estimations of couplings can be further used in specifying the basic Z' model.

Usually, the couplings are considered as independent arbitrary numbers. However, this is not the case and they are correlated parameters, if the basic model is renormalizable one. Hence, correlations follow and the amount of free low energy parameters reduces. Moreover, the correlations between couplings influence kinematics of the processes that gives a possibility for introducing the specific observables which uniquely pick out the virtual Z' boson. The noted additional requirement assumes searching for new particles within the class of renormalizable models. In other aspects the models are not specified. In what follows, we will say "model-independent approach" in the case when either the mass or the couplings must be fitted.

In the present talk we search for the Abelian Z' boson coming from the extended renormalizable model. There are numerous models of such type. In what follows, we say Z' boson for the Abelian one. We also assume, as usually, that the SM is the subgroup of the extended group and therefore no interactions of the type $ZZ'W^+W^-$ appear at a tree-level.

We analyze the deviations of the differential cross-sections for the annihilation process $e^+e^- \to \mu^+\mu^-(\tau^+\tau^-)$ from the SM predictions considered at center-of-mass energies 500 - 1000 GeV. We introduce new observables, $A(E, m_{Z'})$, giving a possibility for estimating both the axial-vector coupling of the Z' to the SM fermions $a_{Z'}$ and the mass $m_{Z'}$, and the observable $V(E, m_{Z'})$, for fitting the products of vector couplings $v_e v_\mu$, $v_e v_\tau$ and the mass $m_{Z'}$.

At ILC energies and expected particle masses, distinguishable properties of the factors at couplings entering the cross-section are observed that gives a possibility for introducing noted observables. Their values can be used in subsequent determination of the basic Z' model. Moreover, the ratio of $A(E, m_{Z'})$ (or $V(E, m_{Z'})$) taken at different energies depends on the $m_{Z'}$, only and may be used as new observables for model-independent estimation of it.

Z' GAUGE BOSONS

At low energies, Z' boson can manifest itself as virtual intermediate state through the couplings to the SM fermions and scalars. Moreover, the Z boson couplings are also modified due to a Z–Z' mixing. Couplings can be described by adding new $\tilde{G}(Z')$-terms to the electroweak covariant derivatives D^{ew} in the Lagrangian [Sirlin(1989),Degrassi(1987)]

$$\begin{aligned} L_f &= i\sum_{f_L} \bar{f}_L \gamma^\mu \left(\partial_\mu - \frac{ig}{2}\sigma_a W_\mu^a - \frac{ig'}{2} B_\mu Y_{f_L} - \frac{i\tilde{g}}{2}\tilde{B}_\mu \tilde{Y}_{f_L}\right) f_L \qquad (1) \\ &+ i\sum_{f_R} \bar{f}_R \gamma^\mu \left(\partial_\mu - ig' B_\mu Q_f - \frac{i\tilde{g}}{2}\tilde{B}_\mu \tilde{Y}_{f_R}\right) f_R, \\ L_\phi &= \left|\left(\partial_\mu - \frac{ig}{2}\sigma_a W_\mu^a - \frac{ig'}{2} B_\mu Y_\phi - \frac{i\tilde{g}}{2}\tilde{B}_\mu \tilde{Y}_\phi\right)\phi\right|^2, \qquad (2) \end{aligned}$$

where summation over all the SM fermions is understood. In these formulas, $g, g', \tilde{g}$ are the charges associated with the $SU(2)_L, U(1)_Y$, and the Z' gauge groups, respectively, σ_a are the Pauli matrices, Q_f denotes the charge of f in positron charge units, Y_ϕ is the $U(1)_Y$ hypercharge, and $Y_{f_L} = -1$ for leptons and 1/3 for quarks. In case of Abelian Z', the $\tilde{Y}_{f_L} = \tilde{Y}_{f_L}\mathrm{diag}(1,1)$ and $\tilde{Y}_\phi = \tilde{Y}_\phi\mathrm{diag}(1,1)$ are diagonal 2×2 matrices with corresponding coupling factors. These generators do not influence the $SU(2)_L$ symmetry.

The Z–Z' mixing angle θ_0 is determined by the coupling $\tilde{Y}_\phi$ as follows

$$\theta_0 = \frac{\tilde{g}\sin\theta_W\cos\theta_W}{\sqrt{4\pi\alpha_{\mathrm{em}}}}\frac{m_Z^2}{m_{Z'}^2}\tilde{Y}_\phi + O\left(\frac{m_Z^4}{m_{Z'}^4}\right), \qquad (3)$$

where θ_0 is the SM Weinberg angle, and α_{em} is the electromagnetic fine structure constant. There are precision constrains on θ_0 value, coming, in particular, from the LEP1 experiments. It is one of the main parameters of the Z' physics.

Below, we will use the Z' couplings to the vector and axial-vector fermion currents defined as

$$v_f = \tilde{g}\frac{\tilde{Y}_{L,f}+\tilde{Y}_{R,f}}{2}, \qquad a_f = \tilde{g}\frac{\tilde{Y}_{R,f}-\tilde{Y}_{L,f}}{2}. \qquad (4)$$

The Lagrangian (1) leads to the following interactions between the fermions and the Z and Z' mass eigenstates:

$$\begin{aligned}
\mathcal{L}_{Z\bar{f}f} &= \frac{1}{2}Z_\mu\bar{f}\gamma^\mu\left[(v_{fZ}^{\mathrm{SM}}+\gamma^5 a_{fZ}^{\mathrm{SM}})\cos\theta_0 + (v_f+\gamma^5 a_f)\sin\theta_0\right]f, \\
\mathcal{L}_{Z'\bar{f}f} &= \frac{1}{2}Z'_\mu\bar{f}\gamma^\mu\left[(v_f+\gamma^5 a_f)\cos\theta_0 - (v_{fZ}^{\mathrm{SM}}+\gamma^5 a_{fZ}^{\mathrm{SM}})\sin\theta_0\right]f, \qquad (5)
\end{aligned}$$

where f is an arbitrary SM fermion state; v_{fZ}^{SM}, a_{fZ}^{SM} are the SM couplings of the Z-boson.

As it occurs, if the extended model is renormalizable, the relations between the couplings hold [Gulov, Skalozub (2000)]:

$$v_f - a_f = v_{f^*} - a_{f^*}, \qquad a_f = T_{3f}\tilde{g}\tilde{Y}_\phi. \tag{6}$$

Here f and f^* are the partners of the $SU(2)_L$ fermion doublet ($l^* = \nu_l, \nu^* = l, q_u^* = q_d$ and $q_d^* = q_u$), T_{3f} is the third component of weak isospin. They also can be derived by imposing the requirement of invariance of the SM Yukawa term with respect to the $\tilde{U}(1)$ gauge transformations. Therefore the relations (6) are independent of the number of scalar field doublets.

The couplings of the Abelian Z' to the axial-vector fermion current have a universal absolute value proportional to the Z' coupling to the scalar doublet. Then, the Z–Z' mixing angle (3) can be determined by the axial-vector coupling. As a result, the number of independent couplings is significantly reduced. Because of the universality we will omit the subscript f and write a instead of a_f.

CROSS-SECTION FOR Z' DETECTIONS

Let us consider the process $e^+e^- \to l^+l^-$ $(l = \mu, \tau)$ with the non-polarized initial and final fermions.

Two classes of diagrams have to be taken into consideration. The first one includes the pure SM graphs. The second class includes heavy Z' boson as the virtual state described by the effective Lagrangian (5) and the scalar particle contributions. We assume that Z' is decoupled and not excited inside loops at the ILC energies. The tree-level diagram $e^+e^- \to Z' \to l^+l^-$ defines a leading contribution to the cross-section. The cross-section includes the contribution of the interference of the SM amplitudes with the Z' exchange amplitude (having the order $\sim a^2, v_f a$) and the squared of the latter one (of the order $\sim a^4, v_f^4$). The last contribution can be neglected at far from resonance energies. The radiative corrections to the Z'-exchange diagram are incorporated in the improved Born approximation.

The deviation of the differential cross-section for the process can be written in the form

$$\Delta\sigma(z) = \frac{d\sigma}{dz} - \frac{d\sigma^{\mathrm{SM}}}{dz} = f_1^{\mu\mu}(z)\frac{a^2}{m_{Z'}^2} + f_2^{\mu\mu}(z)\frac{v_e v_\mu}{m_{Z'}^2} + f_3^{\mu\mu}(z)\frac{a v_e}{m_{Z'}^2} + f_4^{\mu\mu}(z)\frac{a v_\mu}{m_{Z'}^2}. \quad (7)$$

Here, $z = cos\theta$ is the cosine of scattering angle θ. This cross-section accounts for the relations (6) through the known dimensionless functions $f_i(z)$, since the coupling $\tilde{Y}_\phi$ (the mixing angle θ_0) is substituted by the axial-vector coupling a which is universal parameter.

OSERVABLES FOR a^2 AND $m_{Z'}$

Let us investigate the behavior factors $f_i(z)$ assuming that couplings a, v_f have the same order of magnitude.

For definiteness, in Figs. 1, 2 we show the behavior for energy $E = 500$ GeV in the e^+e^- center-of-mass and the mass $m_{Z'} = 2500, 3000$ GeV. Below, we take the ratio $\Gamma_{Z'}/m_{Z'} \sim 0.1$ (the results for narrow resonances are similar at considered energies).

The function $f_1(z)$ is presented as solid line, the $f_2(z)$ is shown as dot-dashed one and the functions $f_{3,4}(z)$ are shown as dashing line. The $f_{3,4}(z)$ coincide The factors $f_{3,4}(z)$ are suppressed by two orders of magnitude as compared to the $f_1(z)$ and $f_2(z)$.

This behavior makes reasonable introducing the integral observable which picks out the contribution coming from the coupling a^2 in Eq. (7).

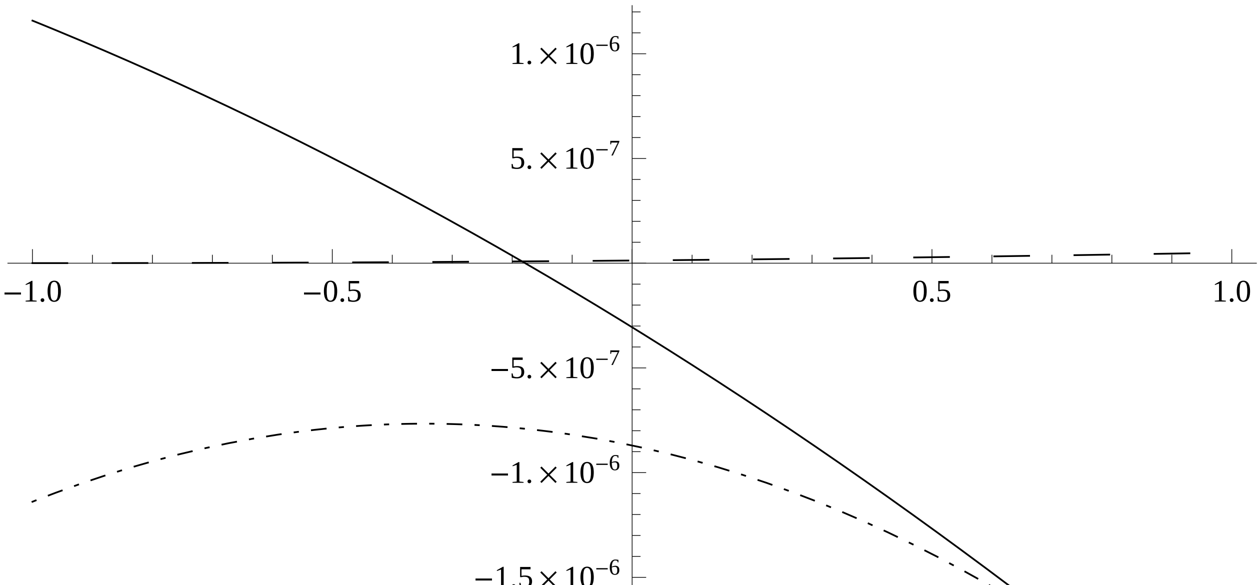

Fig.1 Behavior of factors $f_1(z)$, $f_2(z)$, $f_4(z)$ for $m_{Z'} = 2500$ GeV, width $\Gamma_{Z'} = 250$ GeV for $E = 500$GeV

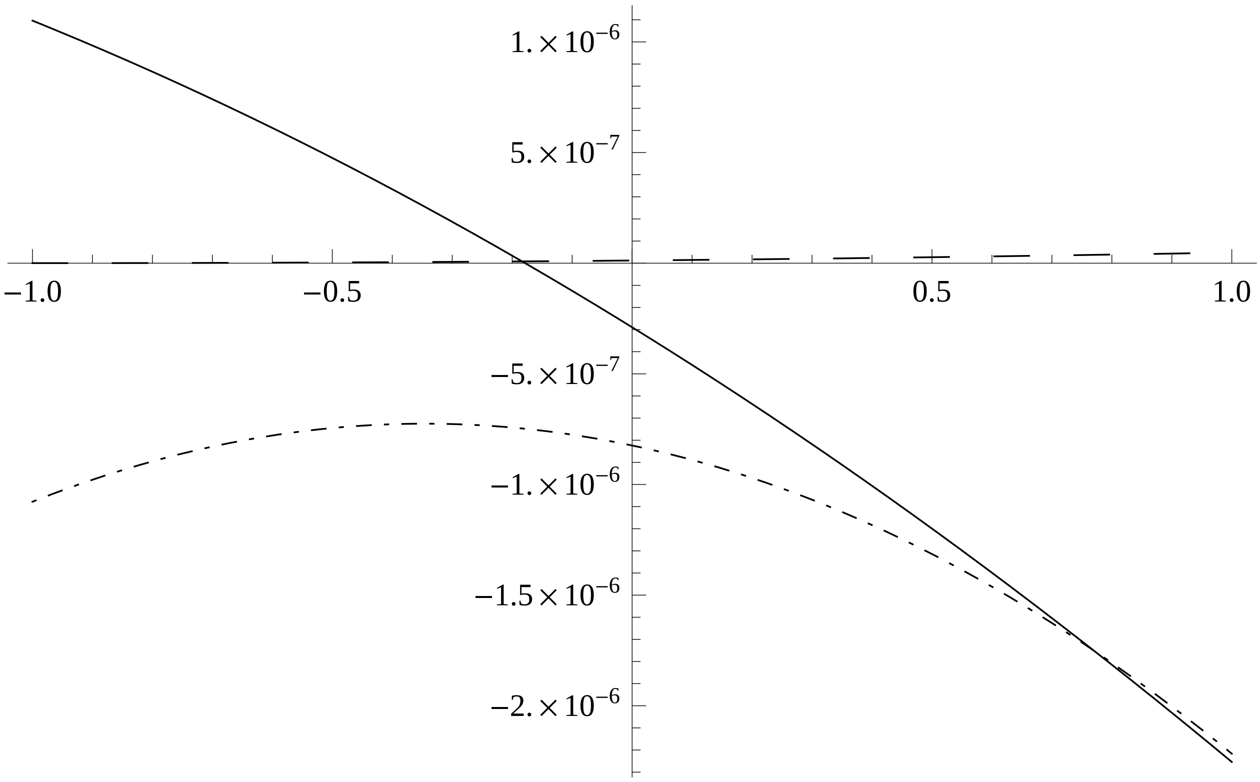

Fig.2 Behavior of factors $f_1(z)$, $f_2(z)$, $f_4(z)$ for $m_{Z'} = 3000$ GeV, width $\Gamma_{Z'} = 150$ GeV for $E = 500$GeV

Really, we can integrate $f_2(z)$ in the intervals $(-1 < z < -0.2)$ (where the function $f_1(z)$ is positive) and $(-0.2 < z < z^*)$ (where $f_1(z)$ is negative) and specify the limit z^* in such a way that the difference of the integrals turns to zero:

$$(\int_{-1}^{-0.2} - \int_{-0.2}^{z^*}) f_2^{\mu\mu}(z) dz = 0. \tag{8}$$

The upper limit of integration equals to $z^* = 0.489$ for a wide interval of both the mass $m_{Z'}$ and beam energies E. It is also important that the function $f_1(z)$ changes its sign at the point $z = -0.2$ for all energies and masses investigated.

On these grounds we introduce the observable for model-independent estimating of the a^2 and $m_{Z'}$:

$$A(E, m_{Z'}) = (\int_{-1}^{-0.2} - \int_{-0.2}^{z*})(\frac{d\sigma}{dz} - \frac{d\sigma^{\rm SM}}{dz}) dz. \tag{9}$$

Here, the lower and upper limits of integration are theoretical bounds. They can be substituted by other ones corresponding to actual set up of experiments.

Table 1: Observable $A(E, m_{Z'})$ for the interval [-0.9,0.406]

Energy	$m_{Z'}$	$\Gamma_{Z'}$	$f_1(z)$	$f_{3,4}(z)$	$f_2(z)$
500	2500	250	$6.98371 \cdot 10^{-7}$	$-7.34139 \cdot 10^{-9}$	$1.1672 \cdot 10^{-9}$
500	3000	300	$6.59927 \cdot 10^{-7}$	$-6.92419 \cdot 10^{-9}$	$-8.61322 \cdot 10^{-11}$
1000	2500	250	$1.51413 \cdot 10^{-6}$	$-1.58729 \cdot 10^{-8}$	$-3.87173 \cdot 10^{-9}$
1000	3000	300	$1.02365 \cdot 10^{-6}$	$-1.67314 \cdot 10^{-8}$	$-2.11544 \cdot 10^{-9}$

We present the results of calculations in the Tables 1 and 2

In first, second and third columns the energy, mass and width values (expressed in GeV) are given, correspondingly. In the fourth column the contribution coming from $f_1^{\mu\mu}(z)$ is adduced. In the fifth and sixth columns the values of the contributions coming from the factors $f_{3,4}^{\mu\mu}(z), f_2^{\mu\mu}(z)$ Eq.(7) are shown.

Table 2: Observable $A(E, m_{Z'})$ for the interval [-0.9,0.406]

Energy	$m_{Z'}$	$\Gamma_{Z'}$	$f_1(z)$	$f_{3,4}(z)$	$f_2(z)$
500	2500	250	$6.98371 \cdot 10^{-7}$	$-7.34139 \cdot 10^{-9}$	$1.1672 \cdot 10^{-9}$
500	3000	300	$6.59927 \cdot 10^{-7}$	$-6.92419 \cdot 10^{-9}$	$-8.61322 \cdot 10^{-11}$
1000	2500	250	$1.51413 \cdot 10^{-6}$	$-1.58729 \cdot 10^{-8}$	$-3.87173 \cdot 10^{-9}$
1000	3000	300	$1.02365 \cdot 10^{-6}$	$-1.67314 \cdot 10^{-8}$	$-2.11544 \cdot 10^{-9}$

$A(E, m_{Z'})$ **is determined by two couplings** a^2 **and** av_μ. The efficiency of the observable is determined from the relation:

$$\kappa_A = \frac{|f_1^{\mu\mu}|}{|f_1^{\mu\mu}| + |f_{3,4}^{\mu\mu}|}. \quad (10)$$

Here the quantities $|f_i^{\mu\mu}|, i = 1, 3, 4,$ mark the integrals

$$(\int_{-1}^{-0.2} - \int_{-0.2}^{z*}) f_i^{\mu\mu}(z) dz > 0. \quad (11)$$

From Tables 1, 2 $\kappa_A = 0.9896$ for all the given energy and mass values.

OSERVABLES FOR ESTIMATION OF $m_{Z'}$

An application of $A(E, m_{Z'})$ (9) for the model-independent determination of the mass $m_{Z'}$.

Consider the ratio $R_A^{experim} = \frac{A(E_1, m_{Z'})}{A(E_2, m_{Z'})}$ of two cross-sections with close energies E_1 and $E_2 = E_1 + \Delta E$ and write

$$R_A^{experim} = \frac{A(E_1, m_{Z'})}{A(E_2, m_{Z'})} = 1 - \frac{\partial \ln A(E_1, m_{Z'})}{\partial E_1} \Delta E. \quad (12)$$

As a theoretical curve R_A^{theory} the function $f_1^{\mu\mu}$ from Eq.(7) has to be substituted in Eq.(12) instead of $A(E_1, m_{Z'})$. As a result, we obtain the observable depending on $m_{Z'}$, only. Hence, the value of the mass can be estimated by means of a standard χ^2 method. The value ΔE can be taken as the difference between the closer beam energies of experiments.

OSERVABLES FOR $v_e v_\mu$ ($v_e v_\tau$)

As we see from the plots and Tables 1, 2, **to exclude the contribution of the a^2-dependent term** we have to integrate the differential cross-section $\Delta\sigma(z)$ (7) over z in the interval $(-1 \leq z \leq z^v)$ and specify the upper limit from the requirement

$$\int_{-1}^{z^v} f_1^{\mu\mu}(z)dz = 0. \tag{13}$$

Hence, we obtain the observable $V_{e\mu}(E, m_{Z'})$ for estimation of $v_e v_\mu$ (or $v_e v_\tau$)

$$V_{e\mu}(E, m_{Z'}) = \int_{-1}^{z^v} (\frac{d\sigma}{dz} - \frac{d\sigma^{\mathrm{SM}}}{dz})dz, \tag{14}$$

where the limit z_v depends on the energy E and mass $m_{Z'}$.

Let us adduce the values of z^v and $V_{e\mu}(E, m_{Z'})$.

Table 3: Upper limit z^v and the value $V_{e\mu}(E, m_{Z'})$

Energy	$m_{Z'}$	$\Gamma_{Z'}$	z^v	$V_{e\mu} \cdot m_{Z'}^2$	$A \cdot V_{e\mu} \cdot m_{Z'}^2$
500	2500	250	0.567466	$-1.50333 \cdot 10^{-6}$	$1.65644 \cdot 10^{-8}$
500	3000	300	0.5675	$-1.42282 \cdot 10^{-6}$	$1.56777 \cdot 10^{-8}$
1000	2500	250	0.570118	$-3.31411 \cdot 10^{-6}$	$3.52717 \cdot 10^{-8}$
1000	3000	300	0.570115	$-2.24064 \cdot 10^{-6}$	$2.38447 \cdot 10^{-8}$

In Table 3, in the fourth column the cosine of boundary angles is adduced. In the last two columns the corresponding values of $V_{e\mu} \cdot m_{Z'}^2$ and the contributions of the factor at the product av_μ are presented.

The efficiency of the observable $V(E, m_{Z'})$ is determined analogously to the κ_A (10) according to the condition

$$\kappa_V = \frac{|f_2^{\mu\mu}|}{|f_2^{\mu\mu}| + |f_{3,4}^{\mu\mu}|}, \tag{15}$$

where $|f_i^{\mu\mu}|, i = 2, 3, 4$, mark the integrals over the interval $-1 < z < z^v$. **The efficiency is estimated as $\kappa_V = 0.9891$.**

The negative sign is also the distinguishable signal of the virtual Z' boson.

The accuracy of possible estimates depends on both theoretical and experimental uncertainties.

The former account for the accuracy of the cross-section calculation, which includes the SM terms and the additional terms coming from the low energy effective Lagrangian Eq.(1). The latter depend on the precision of measurements.

The accuracy of measurements of the introduced observables $A(E, m_{Z'}), V_{e\mu}(E, m_{Z'})$ can be related with the accuracy of measurements of the total cross-section and the forward-backward asymmetry.

As it was estimated for LEP experiments, the theoretical errors is of the order 2 %. So that we assume that not larger values will be for the ILC. At considered energies, the contributions of the omitted terms $\sim a^4, (v_e v_\mu)^2$ are estimated as 0.1 %. According to data in Tables 1-3, the neglected contributions coming from the factors f_3, f_4 are estimated as 1 - 1.5 %. Hence, we estimate the theoretical errors as 3-4 %. The accuracy of measurements of the leptonic cross-sections is expected to be high.

Thus, the couplings and the mass $m_{Z'}$ can be precisely measured either from the differential cross-sections or from data on the total cross-sections.

DISCUSSION

The factors entering the differential cross-section (7) exhibit features giving a possibility for introducing the observables (9) and (14) dependent mainly on only one coupling a^2, or $v_e v_\mu$ $(v_e v_\mu)$, correspondingly, and the mass $m_{Z'}$. So that all these parameters can be estimated within one- or two parameter fits.

We also obtain the discovery reach for Z' with taking into consideration the observable $A(E, m_{Z'})$ (9) and the axial-vector coupling a^2 estimated from the data set of LEP experiments and derived already: $m_{Z'}^{DRA} = 4.4$ TeV.

Next what can be verified is family independence of v_f couplings. The ratio

$$D_v^{\mu\tau} = \frac{V(E, m_{Z'})_\mu}{V(E, m_{Z'})_\tau} = \frac{v_\mu}{v_\tau} \tag{16}$$

depends on the coupling values and has to be unit in the case of the family independence. It can be simply checked.

It is essential that signature of the observables - positive sign of $A(E, m_{Z'})$ and negative sign of $V(E, m_{Z'})$ - is the signal of the Abelian Z' boson.

The found values of the couplings can be compered with the values for the specific renormalizable Z' models. As a result, the number of the perspective candidates can be considerably reduced.