Minimal Length in Quantum Theory and Gravity and Measurability

A.E. Shalyt-Margolin

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and forerunner

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1. *Minimal Length, Infinitesimal Space-Time Quantities and Measurability*

$$l_{min} \propto l_P$$

String theory, loop quantum gravity and so on.

Two approaches

1) l_{min} **-- only for** $\mathrm{E} \approx \mathrm{E}_P$, for $\mathrm{E} \ll \mathrm{E}_P$, $l_{min} \to 0$

2) Provided a minimal length l_{min} exists, it is existent at all the energy scales and not at high (Planck's) scales only.

Measurability

What is inferred on this basis for real physics?

At least, it is suggested that the use of infinitesimal quantities $\boldsymbol{dx_\mu}$ in a mathematical apparatus of both quantum theory and gravity is incorrect.

$$ds^2 = g_{\mu\nu} dx_\mu dx_\nu \qquad (1)$$

that is inexistent because of l_{min}.

Instead of these infinitesimal quantities, reasonable are «minimal variations possible» Δ_{min} of the quantity L having the dimension of length, i.e. the quantity

$$\Delta_{min} L = l_{min}. \qquad (2)$$

However, the «minimal variations possible» of any quantity having the dimensions of length (2) which are equal to $l_{min} \propto l_P$ require, according to the Heisenberg Uncertainty Principle (HUP), maximal momentum $p_{max} \propto P_{Pl}$ and energy $E_{max} \propto E_P$.
Here l_P, P_{Pl}, E_P -- Planck's length, momentum, and energy, respectively.

But at low energies (far from the Planck energy) there are no such quantities and hence in essence $\boldsymbol{\Delta_{min} L = l_{min} \propto l_P}$ (2) corresponds to the high-energy (Planck's) $\propto \boldsymbol{E_P}$ case only.

For the energies lower than Planck's energy, the «minimal variations possible» $\Delta_{min} L$ of the quantity L having the dimensions of length must be greater than l_{min} and dependent on the present E

$$\Delta_{min} \equiv \Delta_{min,E}, \Delta_{min,E} L > l_{min}. \qquad (3)$$

Besides, as we have a minimal length unit l_{min}, it is clear that any quantity having the dimensions of length is «quantized», i.e. its value measured in the units l_{min} equals an integer number and we have

$$L = N_L l_{min}, \text{ ***"Integrality Condition"*** } \qquad (4)$$

where N_L-- positive integer number.

The problem is, how the «minimal variations possible» $\Delta_{min,E}$ (3)
are dependent on the energy or, what is the same,
on the scales of the measured lengths?

GUP (Generalized Uncertainty Principle):

$$\boldsymbol{\Delta x \geq \frac{\hbar}{\Delta p} + \ell^2 \frac{\Delta p}{\hbar},} \qquad \textbf{(GUP)}$$

where $\ell^2 = \alpha' l_p^2$ and α' -- dimensionless numerical factor.
(GUP) leads to the minimal length $l_{min} = \xi l_P = 2\sqrt{\alpha'} l_P$.

$$\boldsymbol{\alpha_x = l_{min}^2 / x^2}, \quad \textbf{(*)}$$

where x is the measuring scale, $l_{min} \propto l_p$ by virtue of GUP.

HUP (Heisenberg Uncertainty Principle):

$$\boldsymbol{\Delta x \geq \frac{\hbar}{\Delta p}} \qquad \textbf{(HUP)}$$

Then assuming that HUP is to a high accuracy derived from GUP
on going to low energies or that HUP is a special case of GUP

at low values of the momentum, we have

$$(\boldsymbol{GUP}, \boldsymbol{\Delta p} \rightarrow \boldsymbol{0}) = (\boldsymbol{HUP}). \qquad (5)$$

By the language of N_L from (5) is nothing else but a change-over to the following:

$$(\boldsymbol{N_{\Delta x}} \approx \boldsymbol{1}) \rightarrow (\boldsymbol{N_{\Delta x}} \gg \boldsymbol{1}). \qquad (6)$$

The assumed equalities in (GUP) and (HUP) may be conveniently rewritten in terms of l_{min} with the use of the deformation parameter $\boldsymbol{\alpha_{\Delta x}}$.

Then with the equality in (GUP) is of the form

$$\Delta x = \frac{\hbar}{\Delta p} + \frac{\alpha_{\Delta x}}{4}\Delta x. \qquad (7)$$

In this case due to formula (7) the equation (11) takes the following form:

$$N_{\Delta x} l_{min} = \frac{\hbar}{\Delta p} + \frac{1}{4N_{\Delta}x} l_{min} \qquad (8a)$$

or

$$(N_{\Delta x} - \frac{1}{4N_{\Delta}x}) l_{min} = \frac{\hbar}{\Delta p}. \qquad (8b)$$

That is

$$\Delta p = \frac{\hbar}{(N_{\Delta x} - \frac{1}{4N_{\Delta}x}) l_{min}}. \qquad (8c)$$

It is easily seen that the parameter $\boldsymbol{\alpha_x}$ is discrete as it is nothing else but

$$\boldsymbol{\alpha_x = l_{min}^2/a^2 = \frac{l_{min}^2}{N_x^2 l_{min}^2} = \frac{1}{N_x^2}}. \qquad (9)$$

It is evident that α_x is irregularly discrete.

At low energies ($\boldsymbol{N_{\Delta x} \gg 1}$), up to a constant

$$\frac{\hbar^2}{l_{min}^2} = \frac{\hbar c^3}{4\alpha' G} \qquad (10)$$

we have

$$\alpha_{\Delta x} = (\Delta p)^2. \qquad (11)$$

But all the above computations are associated with the nonrelativistic case. As known, in the relativistic case, when the total energy of a particle with the mass m and with the momentum p equals:

$$E = \sqrt{p^2 c^2 + m^2 c^4}, \qquad (12)$$

a minimal value for Δx takes the form:

$$\Delta x \approx \frac{c\hbar}{E}. \qquad (13)$$

And in the **ultrarelativistic case**

$$E \approx pc \tag{14}$$

this means simply that

$$\Delta x \approx \frac{\hbar}{p}. \tag{15}$$

Provided the minimal length l_{min} is involved and considering the **"Integrality Condition" (IC)** , in the general case for at the energies considerably lower than the Planck energies $E \ll E_P$ we obtain the following:

$$\begin{gathered}\Delta x = N_{\Delta x} l_{min} \approx \frac{c\hbar}{E}, \\ or \\ \boldsymbol{E} \approx \frac{\boldsymbol{c\hbar}}{\boldsymbol{N_{\Delta x} l_{min}}}.\end{gathered} \tag{16}$$

Similarly, at the same energy scale in the ultrarelativistic case we have

$$p \approx \hbar/(N_{\Delta x} l_{min}). \tag{17}$$

Then

$$(N_{\Delta t} - \frac{1}{4N_{\Delta} t}) t_{min} = \frac{\hbar}{\Delta E}. \tag{18}$$

As shown, for the ultrarelativistic case there is t_{min}.

Next we assume that for **all cases** there is a minimal measuring unit of time

$$t_{min} = l_{min}/v_{max} = l_{min}/c. \qquad (19)$$

Then, we get the **"Integrality Condition" (IC)** for any time t:

$$t \equiv t(N_t) = N_t t_{min}, \qquad (20)$$

for certain $|N_t| \geq 0$ -- integer.
The corresponding energy E

$$E \equiv E(N_t) = \frac{\hbar}{|N_t - \frac{1}{4N_t}| t_{min}}. \qquad (21)$$

Note that at low energies $E \ll E_P$, that is for $|N_t| \gg 1$,
the formula (25) naturally takes the following form:

$$E \equiv E(N_t) = \frac{\hbar}{|N_t| t_{min}} = \frac{\hbar}{|t(N_t)|}. \qquad (22)$$

Definition 1.

1.Let us define the quantity having the dimensions of length ***L*** *or time* ***t*** ***measurable,*** *when it satisfies the relation (4) (and respectively (20)).*

2. Let us define any physical quantity ***measurable****,*
when its value is consistent with point 1) of this Definition.

Thus, **measurable infinitesimal changes** in length (and hence in time) are **impossible** and any such changes are dependent on the existing energies. In particular, a minimal possible **measurable** change of length is l_{min}. It corresponds to some maximal value of the energy E_{max} or momentum P_{max}. If $l_{min} \propto l_P$, then $E_{max} \propto E_P$,$P_{max} \propto P_{Pl}$, where $P_{max} \propto P_{Pl}$, where P_{Pl} is where the Planck momentum.
Then denoting in **nonrelativistic** case with $\Delta_p(w)$ a **minimal measurable** change every spatial coordinate w corresponding to the energy E we obtain

$$\Delta_{P_{max}}(w) = \Delta_{E_{max}}(w) = l_{min}. \qquad (23)$$

Evidently, for lower energies (momentums) the corresponding values of $\Delta_p(w)$ are higher and, as the quantities having the dimensions of length are quantized for $p \equiv p(N_p) < p_{max}$, $\Delta_p(w)$ is transformed to

$$|\Delta_{p(N_p)}(w)| = |N_p| l_{min}. \quad (24)$$

where $|N_p| > 1$-integer so that we have

$$|N_p - \frac{1}{4N_p}| l_{min} = \frac{\hbar}{|p(N_p)|}. \quad (25)$$

In the relativistic case for $E \equiv E(N_E) < E_{max}$

$$|\Delta_{E(N_E)}(w)| = |N_E| l_{min}, \quad (26)$$

where $|N_E| > 1$-integer.

Next we assume that at high energies $E \propto E_P$ there is a possibility only for the **nonrelativistic** case or **ultrarelativistic** case.

Then for the **ultrarelativistic** case:

$$|N_E - \frac{1}{4N_E}| l_{min} = \frac{\hbar c}{E(N_E)} = \frac{\hbar}{|p(N_p)|}, \quad (27)$$

where $N_E = N_p$.
In the relativistic case at low energies we have

$$E \ll E_{max} \propto E_P. \quad (28)$$

And

$$|\Delta_{E(N_E)}(w)| = |N_E| l_{min} = \frac{\hbar c}{E(N_E)}, |N_E| \gg 1 - integer \qquad (29)$$

In the nonrelativistic case at low energies we get

$$|\Delta_{p(N_p)}(w)| = |N_p| l_{min} = \frac{\hbar}{|p(N_p)|}, |N_p| \gg 1 - integer. \qquad (30)$$

In a similar way for the time coordinate t, for $E_{max} \propto E_P$.

$$\Delta_{E_{max}}(t) = t_{min}. \qquad (31)$$

For $E \equiv E(N_t) E_{max}$

$$|\Delta_{E(N_t)}(t)| = |N_t| t_{min}, \qquad (32)$$

where $|N_E| > 1$-integer, so that we obtain

$$|N_t - \frac{1}{4N_t}| t_{min} = \frac{\hbar c}{E(N_t)}. \qquad (33)$$

In the relativistic case at low energies

$$E \ll E_{max} \propto E_P, \qquad (34)$$

and

$$|\Delta_{E(N_t)}(w)| = |N_t| l_{min} = \frac{\hbar c}{E(N_t)}, |N_t| \gg 1 - integer. \qquad (35)$$

Comment 1

Obviously, when l_{min} is involved, the foregoing formulas
for the momentums $p(N_p)$ and for the energies $E(N_E), E(N_t)$
may **certainly** give the highly accurate result
that is close to the experimental one only at the verified low energies:
$|N_p| \gg 1, |N_E| \gg 1, |N_t| \gg 1.$
In the case of high energies $E \approx E_{max} \propto E_P$ or,
what is the same $|N_p| \to 1, |N_E| \to 1, |N_t| \to 1,$
we have a certain, experimentally unverified,
model with a correct low-energy limit.
It should be noted that used dispersion relations (12) are
valid only at low energies $E \ll E_P$. Within the scope
of GUP the high-energy generalization of
--Modified Dispersion Relations (MDRs)--is valid. In what follows,
within the scope of the above definitions,we consider, unless stated otherwise,
only measurable increments (variations) of the space-time quantities
and the corresponding momentums and energies.
Proceeding from all the above, this simply means that
all minimal increments (variations) of the space-time quantities
are dependent on the present energies and coincident
with the corresponding **minimal uncertainties of Uncertainty Principle.**

2. Some Impotrant Examples.

2.1. Example 1. Gravity for the Static Spherically-Symmetric Space With Horizon in Terms of Measurable Quantities.

Now let us consider one example in view of **Definition 1**.
A a static spherically-symmetric horizon in space-time, the horizon being described by the metric

$$ds^2 = -f(r)c^2dt^2 + f^{-1}(r)dr^2 + r^2d\Omega^2. \qquad (40)$$

The horizon location will be given by a simple zero of the function $f(r)$, at the radius $r = a$.
This case is studied in detail by T.Padmanabhan.
(In particular T. Padmanabhan,
Thermodynamical aspects of gravity: new insights,
Reports on Progress in Physics, vol. 73, no. 4, Article ID 046901, 2010.)

It is known that for horizon spaces one can introduce the temperature that can be identified with an analytic continuation to imaginary time.

$$k_B T = \frac{\hbar c f\prime(a)}{4\pi}. \qquad (41)$$

Therewith, the condition $f(\boldsymbol{a}) = 0$ and $f'(a) \neq 0$ must be fulfilled.
Then at the horizon $r = a$ Einstein's field equations

$$\boldsymbol{\frac{c^4}{G}\left[\frac{1}{2}f'(a)a - \frac{1}{2}\right] = 4\pi P a^2} \qquad (42)$$

where $P = T_r^r$ is the trace of the momentum-energy tensor and radial pressure.
$\boldsymbol{a}$ in terms of the corresponding deformation parameter $\boldsymbol{\alpha = \alpha_a}$
"α-representation" of the (42):A. E. Shalyt-Margolin,
***Quantum theory at planck*'s *scale, limiting values, deformed gravity and dark energy problem*,**
International Journal of Modern Physics D, vol. 21, no. 2, Article ID 1250013, 2012:

$$\frac{\boldsymbol{c^4}}{\boldsymbol{G}}\left(-\boldsymbol{f}'(\boldsymbol{\alpha})\boldsymbol{\alpha}^{\mathbf{2}}-\frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{\alpha}\right)=\mathbf{4}\boldsymbol{\pi}\boldsymbol{P}(\boldsymbol{\alpha})\boldsymbol{l}_{\boldsymbol{min}}^{\mathbf{2}}. \qquad (43)$$

Note that in this specific case the parameter α
within constant factors is coincident with the Gaussian curvature $K_a = 1/a^2$
corresponding to a:

$$\frac{l_{min}^2}{a^2} = l_{min}^2 K_a. \qquad (44)$$

We obtain the Einstein equation on horizon, in this case
in terms of the Gaussian curvature

$$\frac{c^4}{G}\left(-f'(K_a)K_a^2 - \frac{1}{2}K_a\right) = 4\pi P(K_a). \qquad (45)$$

This means that up to the constants

$$-f'(K_a)K_a^2 - \frac{1}{2}K_a = P(K_a), \qquad (46)$$

i.e. the Gaussian curvature K_a is a solution of Einstein equations in this case.
Then we examine different cases of the solution (46)
within the scope of **Definition 1** -- in the case when $\boldsymbol{a}$ **is a measurable quantity**.

2.1.1) **Low energies.** $\boldsymbol{a} \gg \boldsymbol{l_{min}}$**.**

Then the radius a is quantized, we have $a = N_a l_{min}$
with the natural number $N_a \gg 1$. Then it is clear that the
Gaussian curvature $K_a = 1/a^2 \approx 0$ takes a (nonuniform) discrete series
of values close to zero, and, within the factor $1/l_{min}^2$, this series
represents inverse squares of natural numbers

$$(K_a) = (\frac{1}{N_a^2}, \frac{1}{(N_a \pm 1)^2}, \frac{1}{(N_a \pm 2)^2}, \dots). \quad (47)$$

Note that $N_a \gg 1$ is associated with the low-energy case $E \ll E_{max} \propto E_P$ and
then as a result, the Gaussian curvature $K_{a_{\pm\Delta_{E(N_E)}}}$ in the case under study
is only slightly different from K_a. ***Thus, the Gaussian curvature K_a, for fixed E, due to its smallness ($K_a \ll 1$) is practically continuously dependent on the increments $\Delta_{E(N_E)}$***

2.1.2) Now we consider the opposite case or the transition to the
ultraviolet limit

$$a = \kappa l_{min}. \quad (48)$$

Here κ is on the order of 1.
In this case the Gaussian curvature K_a is not
a small value continuously dependent on a and a discrete series
of values $K_a, K_{a_{\pm\eta l_{min}}}, K_{a_{\pm\eta' l_{min}}}, ..$, where $\eta, \eta', \dots$-- integers on the order of 1.
In accordance with the above arguments, the limiting transition
to **high energies** $E \approx E_{max} \propto E_P$ gives a discrete chain

of equations or a single equation with a **discrete set of solutions** as follows:

$$-f'(K_a)K_a^2 - \frac{1}{2}K_a = \Theta(K_a)$$

$$-f'(K_{a\pm\eta l_{min}})K_{a\pm\eta l_{min}}^2 - \frac{1}{2}K_{a\pm\eta l_{min}} = \Theta(K_{a\pm\eta l_{min}}) \qquad (49)$$

and so on.
Here $\Theta(K_a)$ -- some function that in the limiting transition to low energies must reproduce the low-energy result to a high degree of accuracy, i.e. $P(K_a)$ appears for $a \gg l_{min}$ from formulae (49)

$$\lim_{K_a \to 0} \boldsymbol{\Theta}(\boldsymbol{K_a})) = \boldsymbol{P}(\boldsymbol{K_a}). \qquad (50)$$

Comment 2.

In this way the above-mentioned example shows that,

despite the absence of infinitesimal spatial-temporal

increments owing to the existence of l_{min} and

the essential "discreteness" of a theory, this discreteness

at low energies is not "felt", the theory being actually continuous.

The indicated discreteness is significant only

in the case of high (Planck) energies.

2.2. Example 2. Heuristic Markov's Model.

M. A. Markov, Ultimate Matter Density as the Universal Low of Nature.
JETP letters., **1982** *36*, 214--216.

It is assumed that «by the universal decree of nature a quantity of the material density ρ is always bounded by its upper value given by the expression that is composed of fundamental constants»:

$$\rho \le \rho_p = \frac{c^5}{G^2 \hbar}, \tag{51}$$

with ρ_p as «Planck's density».

Then the quantity

$$\wp_\rho = \rho/\rho_p \le 1 \tag{51}$$

is the **deformation parameter** to construct the following
of Einstein's equations deformation or $\wp_\rho$-deformation:

$$R_\mu^\nu - \frac{1}{2} R \delta_\mu^\nu = \frac{8\pi G}{c^4} T_\mu^\nu (1 - \wp_\rho^2)^n - \Lambda \wp_\rho^{2n} \delta_\mu^\nu, \qquad (52)$$

where $n \geq 1/2$, T_μ^ν--energy-momentum tensor, Λ-- cosmological constant.
The case of the parameter $\wp_\rho \ll 1$ or $\rho \ll \rho_p$ correlates
with the classical Einstein equation, and
the case when $\wp_\rho = 1$ -- with the de Sitter Universe.
In this way (52) may be considered as $\wp_\rho$-deformation of the General Relativity.
$\wp_\rho$-of Einstein's equations deformation (52)
is nothing else but α-deformation of GR for the parameter $\alpha = \alpha_l$.
If $\rho = \rho_l$ is the average material density for the Universe
of the characteristic linear dimension l, i.e. of the volume $V \propto l^3$, we have

$$\wp_{l,\rho} = \frac{\rho_l}{\rho_p} \propto \alpha_l^2 = \omega \alpha_l^2, \qquad (53)$$

where ω is some computable factor.
Then it is clear that α_l-representation (52) is of the form

$$R_\mu^\nu(\alpha_l) - \frac{1}{2} R(\alpha_l) \delta_\mu^\nu = \frac{8\pi G}{c^4} T_\mu^\nu(\alpha_l) - \Lambda(\alpha_l) \delta_\mu^\nu. \qquad (54)$$

Thus, in this specific case we obtain the explicit dependence of GR on the available energies $E \propto 1/l$, that is insignificant at low energies or for $l \ll l_{min}$ and, on the contrary, significant at high energies, $l \to l_{min}$.

(2.2.1) **Low energies. Nonmeasurable case**.
In this case at low energies, in the limit $l_{min} \to 0$ we get a **continuous theory** coincident with the General Relativity.
(2.2.2) **Low energies. Measurable case**. In this case at low energies, $\alpha_l = 1/{N_l}^2$ (and $N_l \gg 1$), we get a **discrete theory** which is a **«nearly continuous theory»**, practically similar to the General Relativity with the slowly (smoothly) varying parameter $\alpha_{l(t)}$, where t -- time.

So, due to low energies and momentums $(E \ll E_P, p \ll P_{Pl})$, the **«continuous case»** (2.2.1) (General Relativity) and the **«discrete case»** (2.2.2) that is actually a **«nearly continuous case»** are practically indistinguishable in line with **Comment 2** in the preceding Subsection.

(2.3) **At high energies we consider the measurable case** only.
Then it is clear that at high energies the parameter $\alpha_{l(t)}$

is discrete and for the limiting value of $\alpha_{l(t)} = 1$
we get a discrete series of equations
(or a single equation of this form met by a discrete series of solutions)
corresponding to $\alpha_{l(t)} = 1; 1/4; 1/9; \ldots$
In all cases $\Lambda(\alpha_l)$ is not longer a cosmological constant,
being a dynamical cosmological term.

3. Some Consequences, Conclusion and Afterword

3.1. Padmanabhan showed that the Einstein Equation for horizon spaces in the differential form may be written as a thermodynamic identity (the first principle of thermodynamics):

$$\underbrace{\frac{\hbar c f'(a)}{4\pi}}_{k_B T}\ \underbrace{\frac{c^3}{G\hbar} d\left(\frac{1}{4} 4\pi a^2\right)}_{dS}\ \underbrace{-\ \frac{1}{2}\frac{c^4 da}{G}}_{-dE} = \underbrace{Pd\left(\frac{4\pi}{3} a^3\right)}_{P\,dV}. \tag{55}$$

where, as noted above, T -- temperature of the horizon surface,
S --corresponding entropy, E-- internal energy, V -- space volume.
If, the radius of a sphere $r = a$, its entropy:

$$S = \frac{4\pi a^2}{4 l_P^2} = \frac{\pi a^2}{l_P^2}, \tag{56}$$

within the constant factor $1/4l_P^2$ equal to a sphere with the radius a.

Because of this, there are no infinitesimal increments of these quantities, i.e. da, dV, dS. And, provided l_{min} is involved, the Einstein equation for the above-mentioned case in the differential form (55) makes no sense and is useless. If da may be, purely formerly, replaced by l_{min}, then, as the quantity l_{min} is fixed, it is obvious that «dS» and «dV» in (55) will be growing as a and a^2, respectively. And at low energies, i.e. for large values of $a \gg l_{min}$, this naturally leads to infinitely large rather than infinitesimal values. In a similar way it is easily seen that the «Entropic Approach to Gravity» E. Verlinde, **“On the origin of gravity and the laws of Newton,”** *Journal of High Energy Physics*, vol. 2011, no. 4, article 29, 2011 .— in the present formalism is invalid within the scope of the minimal length theory. In fact, the «main instrument» in this paper is a formula for the infinitesimal variation dN in the bit numbers N on the holographic screen S with the radius R and with the surface area A:

$$dN = c^3 G\hbar \, dA = \frac{dA}{l_P^2}. \qquad (57)$$

(57) is meaningless because neither dA nor dN makes sense.

In fact, we have a **no-go theorems**.
The last statements concerning dS, dN may be explicitly interpreted using the language of a quantum information theory as follows:
due to the existence of the minimal length l_{min}, the minimal area l_{min}^2 and volume l_{min}^3 are also involved, and that means «quantization» of the areas and volumes. As, up to the known constants, the «bit number» N and the entropy S from are nothing else but

$$S = \frac{A}{4l_{min}^2}, N = \frac{A}{l_{min}^2}, \qquad (58)$$

it is obvious that there is a «minimal measure» for the «amount of data» that may be referred to as «one bit» (or «one qubit»).
The statement that there is no such quantity as dN (and respectively dS) is equivalent to claiming the absence of $0{,}25$ bit, $0{,}001$ bit, and so on.
This inference completely conforms to the Hooft-Susskind Holographic Principle (HP) that includes two main statements:

(a)All information contained in a particular spatial domain is concentrated at the boundary of this domain.

(b)A theory for the boundary of the spatial domain under study should

contain maximally one degree of freedom per Planck's area l_P^2.

In fact (but not explicitly) HP implicates the existence of $l_{min} = l_P$.
The existence of $l_{min} \propto l_P$ ***totally conforms to HP, providing its generalization.***
Specifically, without the loss of generality, l_P^2 ***in point (b) may be replaced by*** l_{min}^2.

3.2. The main idea is to demonstrate **the existence of the correct limiting high-energy transition**:

$$(\mathbf{2.2.2}) - \boldsymbol{Measurability} \overset{\boldsymbol{High\ Energy}}{\Rightarrow} (\mathbf{2.3}) \qquad (59)$$

and **the nonexistence of the correct limiting high-energy transition**:

$$(\mathbf{2.2.1}) - \mathbf{Non - Measurability} \overset{\boldsymbol{High\ Energy}}{\Rightarrow} (\mathbf{2.3}) \qquad (60)$$

In the general case, based on the parameter α_a this means that **there exists the correct limiting high-energy transition**:

$$\lim_{l_{min}\neq 0, |N_a| \gg 1} \alpha_a \overset{\boldsymbol{High\ Energy}}{\Rightarrow} \lim_{l_{min}\neq 0, |N_a| \approx 1} \alpha_a \qquad (61)$$

and **there is no correct limiting high-energy transition**

$$\lim_{l_{min}=0} \alpha_a \overset{High\ Energy}{\Rightarrow} \lim_{l_{min}\neq 0, |N_a|\approx 1} \alpha_a. \qquad (62)$$

However, the whole theoretical physics, in which presently at low energies $E \ll E_P$ the minimal length l_{min}, is not involved (i. e. $l_{min} = 0$), is framed around the search for **nonexistent limits** (60), (62).

3.3. Basic Result and Aim

3.3.1. When in the theory the minimal length $l_{min} \neq 0$ is actualized (involved) at all the energy scales, a mathematical apparatus of this theory must be changed considerably: no infinitesimal space-time variations (increments) must be involved, the key role being played by the definition of **measurability**.

3.3.2. As this takes place the theory becomes **discrete** at all the energy scales but, as shown by the example in Section 2 and by the considerations of Section 4, at low energies (far from the Planck energies) the sought for theory must be very close in its results to the starting continuous theory (with $l_{min} = 0$). In the process a real **discreteness** is exhibited only at high energies which are close to the Planck energies.

3.3.3. By this approach the theory at low and high energies is associated with a common single set of the parameters $\boldsymbol{N_a}$ or with the dimensionless small parameters $(1/\boldsymbol{N_a} = \sqrt{\alpha_a})$ which are lacking if at low energies the theory is continuous, i.e. when $l_{min} = 0$.

The principal objective of my further studies is to develop for quantum theory and gravity, within the scope of the considerations given above, the corresponding discrete models (with $\boldsymbol{l_{min} \neq 0}$) for all the energy scales and to meet the following requirements:

3.3.4. At low energies the models must, to a high accuracy, represent the results of the corresponding continuous theories.

3.3.5. The models should not have the problems of transition from low to high energies and, specifically, the ultraviolet divergences problem.

The author is hopeful that the correct construction of a low-energy $\mathbf{QFT}^{l_{min}}$ and $\boldsymbol{Grav}^{l_{min}}$ close with the **QFT** to **GR** allows for a more natural transition to quantum (Planck's) gravity.
I.E. $\mathbf{QFT}^{l_{min}}$ **and** $\boldsymbol{Grav}^{l_{min}}$ should offer a rather natural transition from high to low energies

$$[\boldsymbol{N_L} \approx \mathbf{1}] \rightarrow [\boldsymbol{N_L} \gg \mathbf{1}]$$

and vice versa

$$[\boldsymbol{N_L} \gg \mathbf{1}] \rightarrow [\boldsymbol{N_L} \approx \mathbf{1}],$$

where N_L -- integer from formula
(determining the characteristics scale of the lengths L
(energies $E: 1/L \propto 1/N_L$).
Besides, within the notion of **measurability**,
gravity could be saved from some odd solutions, from **wormholes** in particular.

Other example

K.Godel's solution of Einstein Equations:

$$ds^2 = \frac{1}{2\omega^2}(-(\mathrm{dt} + \mathrm{e^x dz})^2 + dx^2 + dy^2 + \frac{1}{2}\mathrm{e^{2x}}dz^2).$$

$$-\infty < \mathrm{t, x, y, z} < \infty, \omega = \mathrm{const} \neq 0$$

**Gödel, K. (1949). «An example of a new type of cosmological solution of Einstein's field equations of gravitation»,
Rev. Mod. Phys. 21: 447–450.**

Rotation Universe -- nonphysical solution

3.4. Afterword

Remark 1.

Also, it should be noted that the lattice is usually understood
as a uniform discrete structure with one and the same
constant parameter $\boldsymbol{a}$ (lattice pitch).
But, in our case we have a nonuniform discrete structure (lattice in its nature),
where the analogous parameter is variable, is a multiple of $\boldsymbol{l_{min}}$
i. e. $\boldsymbol{a = N_a l_{min}}$ lattice pitch , and also is **dependent on the energies**.
Only in the limit of high (Planck's) energies
we get a (nearly) uniform lattice with (nearly) constant pitch $\boldsymbol{a \approx l_{min}}$

Remark 2.

($\boldsymbol{l_{min}}$ is absent or it is not known existence $\boldsymbol{l_{min}}$)
For any coordinate $\boldsymbol{x} \rightarrow \boldsymbol{x} + \Delta x$ **increment** -- $\Delta x = \widehat{\Delta x}$
-- **uncertainty** from the **Uncertainty Principle** $\widehat{\Delta x}\, \widehat{\Delta p} = \frac{1}{2} \hbar$
It is clear then ***small-scale*** $\Delta \boldsymbol{x}$ **gives large** $\widehat{\Delta p}(\widehat{\Delta E})$
and infinitesimal Δx **gives infinitely large** $\widehat{\Delta p}(\widehat{\Delta E})$
Therefore for $E \ll E_p$, Δx **is necessarily limited from below.**