Alternative way to understand the unexpected results of the JLab polarization experiments to measure the Sachs form factors ratio

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The basis of my talk are the results of our joint with professor E.A. Kuraev papers, which were published in [1-4] . Note that our work [1] in the PRD was published online March 5, 2014 i.e. at the next day after professor E.A. Kuraev died.

[1]. M.V. Galynskii, E.A. Kuraev, Alternative way to understand the unexpected results of the JLab polarization experiments to measure the Sachs form factors ratio, Phys. Rev. D, **89**, 054005 (2014).

[2]. M.V. Galynskii, E.A. Kuraev, On the Physical Meaning of Sachs Form Factors and on the Violation of the Dipole Dependence of G_E and G_M on Q^2, JETP Lett. **96**, 6 (2012).

[3]. M. V. Galynskii, E. A. Kuraev, and Yu. M. Bystritskiy, Possible method for measuring the proton form factors in processes with and without proton spin flip, JETP Lett. **88**, 481 (2008).

[4]. M.V. Galynskii, E.A. Kuraev, arXiv: 1210.0634 [nucl-th].

1. Rosenbluth Method or Rosenbluth Technique

In elastic electron proton scattering $e(p_1) + p(q_1) \to e(p_2) + p(q_2)$ there are primarily two methods used to extract the proton form factors. The first method is the Rosenbluth separation method, which uses measurements of the unpolarized cross section and in the laboratory reference frame when $q_1 = (M, \vec{0})$ and $m_e = 0$ in one-photon exchange approximation read as [1]:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 E_2 \cos^2(\theta_e/2)}{4E_1^3 \sin^4(\theta_e/2)} \frac{1}{1+\tau} \left(G_E^2 + \frac{\tau}{\varepsilon} G_M^2 \right) . \quad (1)$$

$$G_E = F_1 - \tau F_2 \, , \; G_M = F_1 + F_2 \, . \quad (2)$$

Here $\tau = Q^2/4M^2$, $Q^2 = -q^2 = 4E_1 E_2 \sin^2(\theta_e/2)$, $q = q_2 - q_1$, $\alpha = 1/137$ - fine structure constant, $\varepsilon^{-1} = 1 + 2(1+\tau)\tan^2(\theta_e/2)$, $\boldsymbol{\varepsilon}$ **is the degree of the linear polarization of the virtual photon** [2-5]!

[1]. M. Rosenbluth, Phys. Rev. **79**, 615 (1950)
[2]. N. Dombey, Rev. Mod. Phys. **41**, 236 (1969).
[3]. A. Akhiezer, M. Rekalo, Fiz.Elem.Chast.Atom.Yadra **4**, 662 (1973).
[4]. M. Galynskii and M. Levchuk, Yad. Fiz. **60**, 2028 (1997).
[5]. M. Galynskii and E. Kuraev, arXiv:1210.0634v2 [nucl-th].

2. Polarization transfer method of Akhiezer and Rekalo

A.I. Akhiezer and M.P. Rekalo proposed a method for measuring the ratio of the Sachs form factors in the reaction $\vec{e}p \to e\vec{p}$ [1,2]. Their method relies on the phenomenon of polarization transfer from the longitudinally polarized initial electron to the final proton and requires measurement of the spin-dependent cross section. This method is called by the polarization transfer or polarized target (PT) method. In papers [1,2] was shown that the ratio of the degrees of longitudinal (P_l) and transverse (P_t) polarizations of the scattered proton has the form

$$\frac{P_l}{P_t} = -\frac{G_M}{G_E}\frac{E_1+E_2}{2M}\tan\frac{\theta_e}{2}\,. \tag{3}$$

[1]. A. Akhiezer, M. Rekalo, DAN SSSR **13**, 572 (1968),
[2]. A. Akhiezer, M. Rekalo, Fiz.Elem.Chast.Atom.Yadra **4**, 662 (1973).

3. The method of PT from the initial to the final proton

In the Diagonal Spin Basis (DSB) the cross section for the elastic process $e\vec{p} \to e\vec{p}$ naturally splits into the sum of two terms containing only the squares of the Sachs form factors and corresponding to the contribution of transition without ($\sim G_E^2$) and with ($\sim G_M^2$) proton spin-flip [1,2]:

$$\sigma_{ep\to ep}^{\delta_1,\delta_2} \sim (1+\delta_1\delta_2) \times W_{ep\to ep}^{\delta,\delta} + (1-\delta_1\delta_2) \times W_{ep\to ep}^{-\delta,\delta} \,, \ \delta_i = \pm 1, \tag{4}$$

$$W_{ep\to ep}^{\delta,\delta} = \frac{4M^2 G_E^2}{q_+^2 q_-^4} \left[(p_+q_+)^2 + q_+^2 q_-^2\right] \,, \ q_\pm = q_2 \pm q_1, \tag{5}$$

$$W_{ep\to ep}^{-\delta,\delta} = \frac{4M^2 \tau\, G_M^2}{q_+^2 q_-^4} [(p_+q_+)^2 - q_+^2(q_-^2 + 4m_e^2)], \ p_+ = p_1 + p_2. \tag{6}$$

DSB spin 4-vectors s_1 and s_2 of the protons read as *(S.Sikach, 1984)*:

$$s_1 = -\frac{(v_1v_2)v_1 - v_2}{\sqrt{(v_1v_2)^2 - 1}} \,, \ \ s_2 = \frac{(v_1v_2)v_2 - v_1}{\sqrt{(v_1v_2)^2 - 1}} \ , v_1 = \frac{q_1}{M}, \ v_2 = \frac{q_2}{M}, \tag{7}$$

In the laboratory system, $q_1 = (M, \vec{0})$, s_1 and s_2 (7) have the form:

$$s_1 = (0, \vec{n}_2), \ s_2 = (|\vec{v}_2|, v_{20}\, \vec{n}_2), \ \ \vec{c}_1 = \vec{c}_2 = \vec{n}_2 = \vec{q}_2/|\vec{q}_2| \,. \tag{8}$$

[1]. M. Galynskii, E. Kuraev, Y. Bystritskiy, JETP Lett. **88**, 481 (2008).
[2]. M.V. Galynskii, E.A. Kuraev, arXiv: 1210.0634 [nucl-th].

Erroneous terminology (red color)

Paper 1. I. A. Qattan, J. Arrington, A. Alsaad, PRC **91**, 065203 (2015),

Citation 1: In electron scattering there are primarily two methods used to extract the proton form factors. The first method is the Rosenbluth or Longitudinal-Transverse (LT) separation method [1], which uses measurements of the unpolarized cross section, and the second is the polarization transfer or polarized target (PT) method [2, 3], which requires measurement of the spin-dependent cross section.

Citation 2: ε is the virtual photon longitudinal polarization parameter...

Paper 2. *A.V. Gramolin, et al., J.Phys. G: Nucl. Part. Phys. 41 (2014) 115001*

$$\frac{d\sigma_{\text{Born}}}{d\Omega_\ell} = \frac{1}{\varepsilon(1+\tau)}\left[\varepsilon\, G_E^2(q^2) + \tau\, G_M^2(q^2)\right]\frac{d\sigma_{\text{Mott}}}{d\Omega_\ell}, \tag{9}$$

Citations:

1. $\varepsilon = \left[1 + 2(1+\tau)\tan^2(\theta/2)\right]^{-1}$ is a dimensionless variable, $0 < \varepsilon < 1$;
2. ε describing the separation between the longitudinal (charge) and transverse (magnetic) parts of the cross section;

[1]. M. Rosenbluth, Phys. Rev. **79**, 615 (1950).
[2]. N. Dombey, Rev. Mod. Phys. **41**, 236 (1969).
[3]. A. Akhiezer, M. Rekalo, Fiz. Elem. Chast.Atom.Yadra **4**, 662 (1973).

The absolutely correct terminology (red color)

I found only one work [4], where the written words about the physical meaning of the variable ε are absolutely correct....

Citation from [4], top of page 5:

"Let us introduce another set of kinematical variables: Q^2,
and the degree of the linear polarization of the virtual photon, ε.

[4] G.I. Gakh, E. Tomasi-Gustafsson, Model independent analysis of polarization effects in elastic electron-deuteron scattering in presence of two-photon exchange.
Nuclear Physics A 799, 127 (2008).

With the aid of Rosenbluth's technique, it was found that the experimental dependences of G_E and G_M on Q^2 are well described up to $5-6$ GeV^2 by the dipole-approximation expression

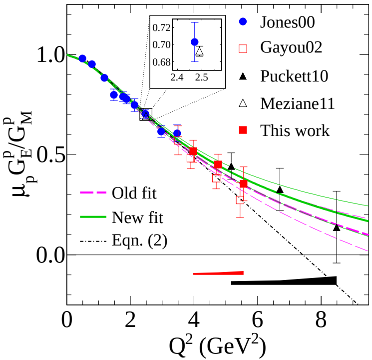
$$G_E = G_M/\mu_p = G_D(Q^2) \equiv \frac{1}{(1+Q^2/0.71)^2} \sim \frac{1}{Q^4}, \;\; \mu_p \frac{G_E}{G_M} \approx 1\,, \tag{10}$$

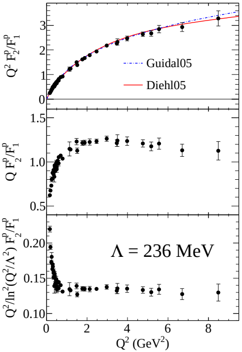
where μ_p is the proton magnetic moment ($\mu_p = 2.79$).

Precision experiments based on employing of the method of Akhiezer and Rekalo were performed at JLab. They showed that, in the range of $0.5 < Q^2 < 5.5$ GeV^2, there was a linear decrease in the ratio $R = \mu_p G_E/G_M$ with increasing Q^2:

$$R \equiv \mu_p G_E/G_M = 1 - 0.13\,(Q^2 - 0.04)\,, \tag{11}$$

which indicates that G_E falls faster than G_M. In the non-relativistic limit, this fact could be interpreted as indicating that the spatial distributions of charge and magnetization currents in the proton are definitely different.





Polarization transfer data for G_E^p/G_M^p, Q^2F_2/F_1 and QF_2/F_1 from [1]. The linear fit of equation (12) is shown for comparison.

$$R \equiv \mu_p G_E/G_M = 1 - 0.13\,(Q^2 - 0.04)\,. \tag{12}$$

$$Q^2 F_2/F_1 \sim (\log^2 Q^2/\Lambda^2) \neq \text{const}.$$

[1]. A. Puckett *et al.*, PRC, **85** (2012) 045203.
[2]. A. Belitsky, X. Ji, and F. Yuan, PRL **91**, 092003 (2003).
[3]. J. Ralston, PRD **69**, 053008 (2004), Endpoint-overlap model.

The rapid decrease of G_E should not have been a complete surprise, as it had been predicted in at least 4 papers [19-22].

[19] D. V. Volkov, JETP Lett. **2**, 181 (1965).
[20] F. Iachello *et al.* Phys. Lett. **B 43**, 191 (1973).
[21] G. Holzwarth, Z. Phys. **A 356**, 339 (1996).
[22] M. Frank *et al.* Phys. Rev. **C 54**, 920 (1996).

In order to resolve this contradiction, it was assumed that the discrepancy in question may be caused by disregarding, in the respective analysis, the contribution of two-photon exchange (TPE):

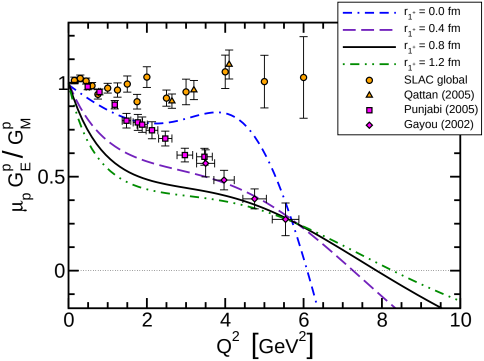
[P. Guichon and M. Vanderhaeghen, Phys. Rev. Lett. **91**, 142303 (2003)].
[N. Kivel and M. Vanderhaeghen, Phys. Rev. Lett. **103**, 092004 (2009)]
[J. R. Arrington *et al.*, Prog. Part. Nucl. Phys. **66**, 782 (2011)].

At the present time, three experiments aimed at studying the contribution of TPE are known:

1) experiment at the VEPP-3 storage ring in Novosibirsk (+),
2) the EG5 CLAS experiment at JLab (+),
3) the OLYMPUS experiment at the DORIS accelerator at DESY (-).

I. C. Cloët *et al.*, Survey of Nucleon Electromagnetic Form Factors, Few Body Syst. **46**, 1-36 (2009).

In this work was shown that nonperturbative effects are important !!! :-)



Result for the normalised ratio of Sachs electric and magnetic form factors. Data: diamonds – Gayou2000; squares – Punjabi2005; triangles – Qattan2004; and circles – Walker1993.

Where does the pQCD behavior begin?

It is, in general, admitted that the onset of the asymptotic regime of pQCD starts around the J/Ψ mass squared, i.e. at $Q^2 \approx 9.0$ GeV^2. It was first observed in work R. Arnold *et al.*, PRL **57**, 174 (1986) that G_M, follows the asymptotic pQCD predictions of Lepage and Brodsky, PRD **22**, 2157 (1980), and $Q^4 G_M$ becomes nearly constant starting at $Q^2 \approx 9$ GeV^2. The answer to the question what is in general admitted at present on the onset of pQCD can be found in [1,2]:

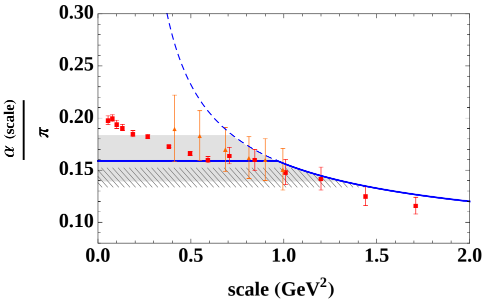
[1] A. Courtoy and S. Liuti, Phys.Lett.B **726**, 320 (2013).

[2] S. Brodsky *et al.*, Phys.Rev.D **81**, 096010 (2010).

In this works based on using completely different approaches, it is shown that the point of transition from non-perturbative QCD to pQCD correspond to a momentum scale $Q_0 \sim 1$ GeV. For this reason we will below assume that HSM of pQCD starts at the lower boundary of the considered region, i.e. around $Q_0 \sim 1$ GeV.

Where does the pQCD behavior begin?

[1] A. Courtoy and S. Liuti, Phys.Lett.B **726**, 320 (2013).



Extraction of α_s. The blue dashed curve represents the exact NLO solution for the running coupling in $\overline{MS}$ scheme. The solid blue curve represents the running coupling obtained from our analysis using inclusive electron scattering data at large x. Owing to large x resummation, at lower values of the scale, $\alpha_s = \alpha_{s,NLO}$ (min) is frozen as explained in the text. The grey area represents the region where the freezing occurs for JLab data, while the hatched area corresponds the freezing region determined from SLAC data.

Where does the pQCD behavior begin?

[3] R. Pasechnik, D. Shirkov and O. Teryaev, PRD **78**, 071902 (2008).

In [3], within the analytic perturbation theory (APT) approach using the rules of the Gerasimov-Drell-Hearn, it is shown that the point of "crosslinking" of the perturbative and nonperturbative regimes in APT is significantly lower than that obtained in the framework of the standard pQCD, where $Q_0 \sim 1$ GeV. The main reason for such a significant forwarding down of Q within the APT approach is the disappearance of the nonphysical singularities of the perturbation theory series.

It should be noted that in the known work of Belitsky *et al.* [4] the authors have performed numerical calculations in the framework of pQCD in the region of $0.5 \leq Q^2 \leq 5.5$ GeV^2; therefore, they proceeded from the assumption that the onset of pQCD starts already at $Q^2 = 0.5$ GeV^2.

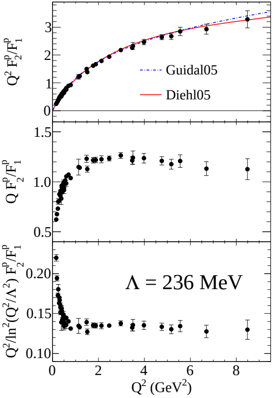
It is very likely that the results of Ref. [4] are an indirect proof of the correctness results of Ref. [3] obtained in the framework of the APT.

[4] A. Belitsky, X. Ji, and F. Yuan, PRL **91**, 092003 (2003).

Where does the pQCD behavior begin?

In work [4] authors derive at the logarithmic accuracy the asymptotic scaling which describes recent Jefferson Lab data well.

$$Q^2 F_2/F_1 \sim (\log^2 Q^2/\Lambda^2) \neq \text{const.} \qquad (13)$$



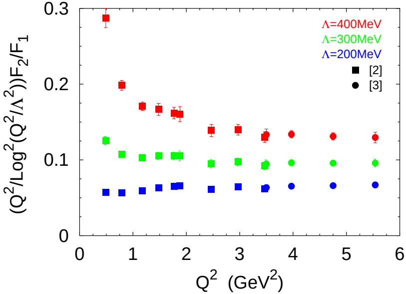
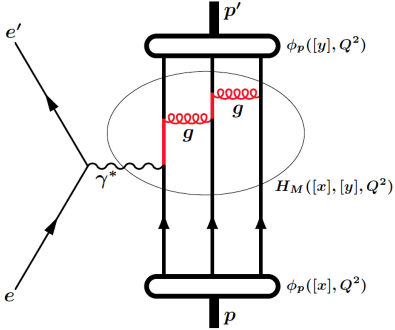


Fig. 3 from Ref. [4] (right figure). JLab data plotted in terms of the leading PQCD scaling. The low, middle, and upper data points correspond to $\Lambda = 200, 300, 400$*, respectively.*

[4] A. Belitsky, X. Ji, and F. Yuan, PRL **91**, 092003 (2003).

What is the hard-scattering mechanism of pQCD?

G. Lepage and S. Brodsky, Phys. Rev. D **22**, 2157 (1980).

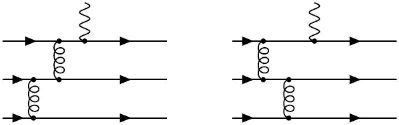


A typical hard gluon-exchange process in elastic electron-proton scattering $(e + p \to e' + p')$. *There are two hard quark propagators and two gluon ones which contribute to the counting rule in the elastic form factor.*

The aims of this paper are:
(i) the interpretation in the one-photon exchange approximation unexpected results of the JLab polarization experiments to measure the Sachs FFs ratio as well as the explanation of the reason for the linear dependence in (11),
(ii) the determination of the conditions for the realization of the Sachs FFs dipole dependence based on the use of the hard-scattering mechanism (HSM) of perturbative QCD (pQCD) under the assumption that the onset of pQCD starts around the lower boundary of the region $1.0 \leq Q^2 \leq 8.5$ GeV^2 of the experimental measurements.
For this purpose, we developed an approach which essentially is a generalization of the constituent-counting rules of the pQCD for the case of massive quarks. Abandoning the massless quarks, we were able to explain in the one-photon exchange approximation the unexpected results of measurements of the proton Sachs FFs ratio and analytically derive the experimentally established formula of the linear decrease law for this ratio at $\tau < 1$.
Our interpretation can be considered as a possible way to solve the G_E/G_M problem.

On the dependence G_E and G_M on Q^2

Let us consider the HSM of pQCD in the process $ep \to ep$ that is realized as we believe at $Q^2 \geq 1$ GeV^2. In this case the leading contribution to the proton current $J_p^{\pm\delta,\delta}$ can be presented as a sum of the hard gluon exchange processes, where the proton is replaced by a set of three almost on mass shell quarks as illustrated in figure below



Below we will suppose the masses of quarks m_q to be equal to $1/3$ of the proton mass M and the fraction of their transfer momenta to be equal. So we have

$$\tau_q = \tau_p = \tau = Q^2/4M^2 \,. \tag{14}$$

Under such simplifying assumptions it can easily be verified that the matrix element corresponding to the sum of two gauge-invariant diagrams, shown in this figure, has the form

$$(J_{p_{1,2}}^{\pm\delta,\delta})^\mu \sim (J_q^{\pm\delta,\delta})^\nu (J_q^{\pm\delta,\delta})_\nu (J_q^{\pm\delta,\delta})^\mu / Q^6 \,. \tag{15}$$

The absolute magnitudes of the proton current matrix elements $J_p^{\pm\delta,\delta}$ that correspond to the contribution of the full set of possible Feynman diagrams can be written as the product of three point-quark current amplitudes $J_q^{\pm\delta,\delta}$ (27) divided by Q^6,

$$J_p^{\pm\delta,\delta} \sim J_q^{\pm\delta,\delta} J_q^{\pm\delta,\delta} J_q^{\pm\delta,\delta} / Q^6 \,. \tag{16}$$

There are two possibilities for a proton non-spin-flip transition: (i) none of the three quarks undergoes a spin-flip transition and (ii) two quarks undergo a spin-flip transition, while the third does not. We denote the number of such ways as $n_{qE}^{-\delta,\delta} = [0,2]$. Proton spin-flip can also proceed in two ways: (i) one quark undergoes a spin-flip transition, while the other two do not, and (ii) all three quarks undergo a spin-flip transition. We denote the number of such ways by $n_{qM}^{-\delta,\delta} = [1,3]$. Thus, there are in all four combinations to be considered:

$$n_{qE}^{-\delta,\delta} \times n_{qM}^{-\delta,\delta} = (0,1) \oplus (0,3) \oplus (2,1) \oplus (2,3) \,. \tag{17}$$

Note due to Eqs. (27), (14) at $\tau \ll 1$ ($\tau \gg 1$) the quark transition without (with) spin-flip dominates. Therefore, the sets (0,1) and (2,3) are realized at $\tau \ll 1$ and $\tau \gg 1$, respectively.

The matrix elements of the proton current in the DSB

Let us consider the process of elastic ep scattering

$$e(p_1) + p(q_1, s_1) \to e(p_2) + p(q_2, s_2)\,, \tag{18}$$

where s_1 and s_2 are spin 4-vectors for initial and final protons.Matrix elements for proton currents were calculated in DSB by S.Sikach (1984):

$$(J_p^{\pm\delta,\delta})_\mu = \overline{u}^{\pm\delta}(q_2)\Gamma_\mu(q^2)u^\delta(q_1)\,, \;\; \Gamma_\mu(q^2) = F_1\,\gamma_\mu + \frac{F_2}{4M}(\,\hat{q}\gamma_\mu - \gamma_\mu\hat{q}\,)\,.$$

$$(J_p^{\delta,\delta})_\mu = 2\,M\,G_E\,(b_0)_\mu\,,\;(J_p^{-\delta,\delta})_\mu = -2\delta M\sqrt{\tau}\,G_M\,(b_\delta)_\mu,\; b_\delta = b_1 + i\delta\,b_2\,. \tag{19}$$

For the point quarks with mass m_q the amplitude of the currents read as

$$(J_q^{\delta,\delta})_\mu = 2\,m_q\,(b_0)_\mu\,,(J_q^{-\delta,\delta})_\mu = -2\,m_q\,\delta\sqrt{\tau_q}\,(b_\delta)_\mu\;\;,\tau_q = Q_q^2/4m_q^2\,. \tag{20}$$

4-vectors b_0, b_1, b_2, b_3 is an orthonormal basis of 4-vectors:

$$b_0 = \frac{q_+}{\sqrt{q_+^2}}\,,b_3 = \frac{q_-}{\sqrt{-q_-^2}},\;(b_1)_\mu = \varepsilon_{\mu\nu\kappa\sigma}b_0^\nu b_3^\kappa b_2^\sigma,\;(b_2)_\mu = \varepsilon_{\mu\nu\kappa\sigma}b_0^\nu b_3^\kappa p_1^\sigma/\rho\,, \tag{21}$$

where $,\, q_+ = q_2 + q_1,\; q_- = q_2 - q_1,\; \varepsilon_{\mu\nu\kappa\sigma}$ is the Levi-Civita tensor.

Rosenbluth formula in the arbitrary reference frame

With the help of the matrix elements of the proton current (19), calculation probability of the process $ep \to ep$ is reduced to trace:

$$| T |^2 = \frac{4M^2}{q^4}\,\frac{1}{8}\sum_{\delta} Tr(G_E^2(\hat{p}_2+m)\hat{b}_0(\hat{p}_1+m)\hat{b}_0 + + \tau\, G_M^2(\hat{p}_2+m)\hat{b}_\delta(\hat{p}_1+m)\hat{b}_\delta^*)\ , \tag{22}$$

where $b_\delta^* = b_{-\delta} = b_1 - i\delta b_2, \delta = \pm 1$.

$$d\sigma = \frac{\alpha^2 do}{4w^2}\,\frac{1}{1+\tau}\,(\ G_E^2\ Y_I + \tau\ G_M^2\ Y_{II}\ )\,\frac{1}{q^4}\ , \tag{23}$$

$$Y_I = (p_+q_+)^2 + q_+^2 q^2, \qquad Y_{II} = (p_+q_+)^2 - q_+^2(q^2+4m_e^2)\ ,$$

$$p_+ = p_1 + p_2\ ,\ q_+ = q_1 + q_2\ .$$

Thus, the differential cross section for the $ep \to ep$ process naturally splits into the sum of two terms containing only the squares of the Sachs FFs and corresponding to the contribution of transition without ($\sim G_E^2$) and with ($\sim G_M^2$) proton spin-flip.

On the dependence G_E and G_M on Q^2

[M.Galynskii, E.Kuraev, Phys. Rev D **89**, 054005 (2014)]

$$(J_p^{\delta,\delta})_\mu = 2\,M\,G_E\,(b_0)_\mu\,,\ (J_p^{-\delta,\delta})_\mu = -2\delta M\sqrt{\tau}\,G_M\,(b_\delta)_\mu,\ b_\delta = b_1 + i\delta\,b_2\ .$$

Since $|b_0| = 1$ and $|b_\delta| = \sqrt{2}$ and they are does not depend on Q^2, then from the (19), we can obtain the dependence on Q^2 for (absolute) values of the amplitudes of the proton currents $J_p^{\pm\delta,\delta}$ and point quarks $J_q^{\pm\delta,\delta}$:

$$J_p^{\delta,\delta} = 2\,M\,G_E,\ J_p^{-\delta,\delta} = 2\,M\,\sqrt{\tau}\,G_M\,, \qquad (24)$$

$$J_q^{\delta,\delta} = 2\,m_q\,,\ J_q^{-\delta,\delta} = 2\,m_q\,\sqrt{\tau_q}\ . \qquad (25)$$

Note that the factorization of $2M$ and $2m_q$ in the expressions (24), (25) is caused by the normalization bispinors $\bar{u}_i u_i = 2m_i$. Below during the computation is more convenient to use the normalization of $\bar{u}_i u_i = 1$, and instead of (24), (25) we will use the expressions:

$$J_p^{\delta,\delta} = G_E, J_p^{-\delta,\delta} = \sqrt{\tau}\,G_M\,, \qquad (26)$$

$$J_q^{\delta,\delta} = 1\,,\quad J_q^{-\delta,\delta} = \sqrt{\tau}\ . \qquad (27)$$

Expressions (26), (27) we will be used below to explain the dependence FFs G_E and G_M on Q^2.

The set (0,1), $G_E, G_M \sim 1/Q^6$, $G_E/G_M \sim 1$

Let us consider the first (0,1) set. We use for the amplitudes of protons and point-like quarks currents expressions (26):

$$J_q^{\delta,\delta} = 1\,, \quad J_q^{-\delta,\delta} = \sqrt{\tau}\,.$$

The matrix elements of the proton current $J_p^{\delta,\delta}$ and $J_p^{-\delta,\delta}$ must be proportional to G_E and G_M, respectively; as a result, we have

$$J_p^{\delta,\delta} = G_E \sim 1 \times 1 \times 1\,/\,Q^6\,, \tag{28}$$

$$J_p^{-\delta,\delta} = \sqrt{\tau}\,G_M \sim \sqrt{\tau} \times 1 \times 1/\,Q^6\,, \tag{29}$$

where the factors of unity and $\sqrt{\tau}$ on the right-hand side of Eqs. (28) and (29) correspond to non-spin-flip transitions for three pointlike quarks and to the spin-flip transition for one quark. As a result, we have

$$G_E \sim \frac{1}{Q^6},\, G_M \sim \frac{1}{Q^6}\,,\, \frac{G_E}{G_M} \sim 1\,. \tag{30}$$

Therefore, for the set (0,1) the FFs ratio G_E/G_M behaves in just the same way as in the dipole case. However, the dependencies $G_E \sim 1/Q^6$, $G_M \sim 1/Q^6$ are not dipole ones.

Let us consider the (0,3) set. We use for the amplitudes of protons and point-like quarks currents expressions (26):

$$J_q^{\delta,\delta} = 1\,, \quad J_q^{-\delta,\delta} = \sqrt{\tau}\,.$$

For this purpose we write equalities similar to (28) and (29); that is,

$$J_p^{\delta,\delta} = G_E \sim 1 \times 1 \times 1\,/\,Q^6\,, \tag{31}$$

$$J_p^{-\delta,\delta} = \sqrt{\tau}\,G_M \sim \sqrt{\tau} \times \sqrt{\tau} \times \sqrt{\tau}/\,Q^6\,. \tag{32}$$

From here, we obtain

$$G_E \sim \frac{1}{Q^6}\,, \quad G_M \sim \frac{\tau}{Q^6}\,, \quad \frac{G_E}{G_M} \sim \frac{1}{\tau} \sim \frac{4M^2}{Q^2}\,, \tag{33}$$

$$Q^2\,\frac{G_E}{G_M} \sim 4M^2 = \text{const.} \tag{34}$$

Relation (34) is sometimes called in the literature the Brodsky saturation law; it really corresponds to a maximal possible number of the quark spin-flip transitions.

The set (2,1), $G_E \sim 1/Q^4, G_M \sim 1/Q^6$

Let us now consider the (2,1) spin combination in the set (17). It is generated by spin-flip transitions for two quarks in the case of the contribution to $J_p^{\delta,\delta}$ and by spin-flip transitions for only one quark in the case of the contribution to $J_p^{-\delta,\delta}$. Following the same line of reasoning as above, one can readily show that, for the (2,1) set, G_E and G_M have the form

$$G_E \sim \frac{\tau}{Q^6}\ ,\ G_M \sim \frac{1}{Q^6}\ ,\ \frac{G_E}{G_M} \sim \tau \sim \frac{Q^2}{4M^2}\ , \tag{35}$$

$$Q^2\ \frac{G_M}{G_E} \sim 4M^2 = \text{const.} \tag{36}$$

The set (2,3), $G_E, G_M \sim 1/Q^4, G_E/G_M \sim 1$

For the set (2,3) we have

$$J_p^{\delta,\delta} = G_E \sim \sqrt{\tau} \times \sqrt{\tau} \times 1 \,/\, Q^6 \;, \tag{37}$$

$$J_p^{-\delta,\delta} = \sqrt{\tau}\, G_M \sim \sqrt{\tau} \times \sqrt{\tau} \times \sqrt{\tau} \,/\, Q^6 \;. \tag{38}$$

Hence, we obtain

$$G_E \sim \frac{1}{Q^4},\; G_M \sim \frac{1}{Q^4},\; \frac{G_E}{G_M} \sim 1 \;. \tag{39}$$

Therefore, the dipole dependence in the behavior of the FFs G_E and G_M on Q^2 occurs in the set (2,3) at $\tau \gg 1$ in the case when a number of quark transitions with spin-flip saturation takes place.

Thus, our approach is in fact a generalization of constituent-counting rules for the massive quarks. Note, in [1] to estimate the leading contribution of the HSM in the proton magnetic FF within the standard pQCD with massless quarks, a method similar to our approach was used. At the same time, formulas (16), (17) in [1] and our formulas (38), are the same and reproduce the well-known result obtained in the works of Brodsky within the framework of the constituent-counting rules before the development of QCD.

[1] H. Kawamura *et al.*, Phys. Rev. D **88**, 034010 (2013)

Spin Parametrization for G_E/G_M

The non-spin-flip and spin-flip proton-current amplitudes ($J_p^{\delta,\delta}$ and $J_p^{-\delta,\delta}$) can be represented as the linear combinations

$$J_p^{\delta,\delta} = \alpha_0 \, J_q^{\delta,\delta} J_q^{-\delta,-\delta} J_q^{\delta,\delta} + \alpha_2 \, J_q^{-\delta,\delta} J_q^{\delta,-\delta} J_q^{\delta,\delta}, \quad (40)$$

$$J_p^{-\delta,\delta} = \beta_1 J_q^{-\delta,\delta} J_q^{\delta,\delta} J_q^{-\delta,-\delta} + \beta_3 \, J_q^{-\delta,\delta} J_q^{\delta,-\delta} J_q^{-\delta,\delta}, \quad (41)$$

where the coefficients α_0, α_2, β_1, and β_3 have a clear physical meaning that is determined by their indices. From Eqs. (40) and (41), we have

$$\frac{G_E}{G_M} = \frac{\alpha_0 \; + \alpha_2 \, \tau}{\beta_1 \; + \beta_3 \, \tau} \, . \quad (42)$$

This expression may serve as a basis for constructing spin parametrization and fits experimental data obtained by measuring the ratio G_E/G_M.
We showed above that at $\tau \ll 1$ the quark transition without spin-flip dominates; the set (0,1) with the minimal number of spin-flip quarks, where $G_E/G_M \sim 1$, must occur. In this case the coefficients α_0 and β_1 in Eq. (42) must have the values close to unity. With allowance for this comment, we expand the right-hand side of (42) in a power series for τ. As a result, we get the law of a linear decrease in the ratio $R = G_E/G_M$ as Q^2 increases,

$$R \approx 1 - (\beta_3 - \alpha_2) \, \tau \, . \quad (43)$$

Conclusion

In the one-photon exchange approximation we discuss questions related to the interpretation of unexpected results of the JLab polarization experiments to measure the Sachs form factors ratio G_E/G_M in the region $1.0 \leq Q^2 \leq 8.5$ GeV^2. For this purpose, we have considered the process of the electron proton elastic scattering within the hard scattering mechanism (HSM) of the pQCD, on the assumption that i) HSM of the pQCD starts at the lower boundary of the considered region, i.e. around $Q_0 \sim 1$ GeV, ii) masses of the all quarks in the proton are equal to 1/3 of the proton mass, and iii) the fraction of their transfer momenta to be equal. Developed by us an approach is essentially a generalization of the constituent-counting rules of the pQCD for the case of massive quarks. This allows us to state that

(i) around $Q_0 \sim 1$ GeV the leading scaling behavior of the Sachs FFs has the form $G_E, G_M \sim 1/Q^6, G_E/G_M \sim 1$, but it is not dipole dependence,

(ii) since for quarks $J_q^{\delta,\delta} \sim 1$ and $J_q^{-\delta,\delta} \sim \sqrt{\tau}$, then the dipole dependence ($G_E, G_M \sim 1/Q^4$) is realized in the asymptotic regime of pQCD when $\tau \gg 1$ when the quark transitions with spin-flip dominate,

Conclusion

(iii) the asymptotic regime of pQCD in the JLab experiments has not yet been achieved, and it is likely that the asymptotic regime for G_E occurs at higher values Q^2 than for G_M,

(iv) the linear decrease of the ratio G_E/G_M at $\tau < 1$ is due to additional contributions to $J_p^{\delta,\delta}$ by spin-flip transitions of two quarks and an additional contribution to $J_p^{-\delta,\delta}$ by spin-flip transitions of three quarks,

(v) one of our predictions is the realization (restoration) of a dipole dependence of the Sachs form factors and the value $R = 1$ for higher values of Q^2 (at $\tau \gg 1$).

Thus, abandoning the massless quarks, we were able to explain in the one-photon exchange approximation the unexpected results of measurements of the proton Sachs FFs ratio and analytically derive the experimentally established formula of the linear decrease law for this ratio at $\tau < 1$.

We believe that the interpretation presented above can be considered as a possible way to solve the G_E/G_M problem.

THANK FOR YOUR ATTENTION