Taking into account of hard photons in MOLLER experiment

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Møller scattering

The **electron-electron scattering**

$$e^- e^- \to e^- e^-$$

named after C. Møller (Annalen der Physik **406**, 531 (1932)) is **straightforward and clear** process.

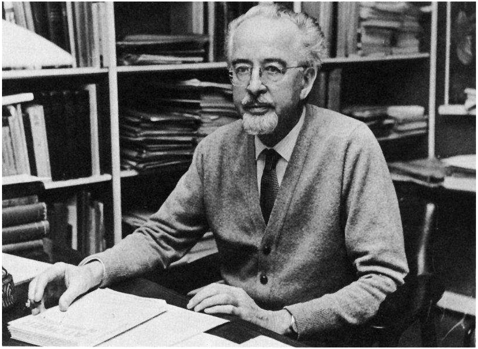

Рис. 1: Christian Møller (1904–1980) was a Danish chemist and physicist who made fundamental contributions to the QED, theories of relativity, gravitation and quantum chemistry.

Motivation

There are several reasons why **polarized** Møller scattering has attracted so much interest of the experimental and theoretical communities:

1. to measure the electron beam polarization (polarimetry):
   - SLC, E-143, E-154 in SLAC,
   - CEBAF beam in JLab,
   - MIT-Bates, and COMPASS in CERN;
2. to determine $\sin\theta_W$ with extra accuracy from the measurement of the parity-violating (PV) asymmetry:
   - E-158 in SLAC,
   - MOLLER in JLab;
3. to test the Standard Model, to reveal traces of new physics:
   - MOLLER in JLab,
   - e^-e^- option of ILC/CLIC,
   - $\mu^-\mu^-$-collider.

MOLLER and E-158, its predecessor

Setups of E-158 and MOLLER are similar: longitudinally polarized electron beam scatters off unpolarized electrons in a hydrogen target.

	laboratory	energy, GeV	$\sin^2\theta_W$	stat.	syst.
E-158	SLAC	45–48	0.2397	± 0.0010	± 0.0008
MOLLER	JLab	11–12		± 0.00026	± 0.00013

MOLLER is **M**easurement **O**f a **L**epton **L**epton **E**lectroweak **R**eaction

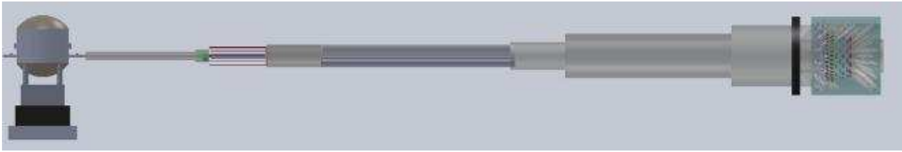

Рис. 2: Layout of the target, spectrometer and detectors for MOLLER.

Polarization PV asymmetry

Observable quantity is a **single-polarization PV asymmetry** for the scattering of longitudinally (Left or Right) polarized electrons on unpolarized (0) ones:

$$A_1 = \frac{\sigma_{L0} - \sigma_{R0}}{\sigma_{L0} + \sigma_{R0}}, \quad \sigma \equiv \frac{d\sigma}{dc}, \quad c = \cos\theta, \tag{1}$$

where θ is the scattering angle of the detected electron in the center of mass (CM) system.

At Born level this asymmetry at low energies $\sqrt{s} \ll m_W = 80.398$ GeV is given by

$$A_1^0 = \frac{s}{m_W^2} \frac{\sin^2\theta}{(3+\cos^2\theta)^2} \frac{1 - 4\sin\theta_{\mathrm{W}}^2}{\sin\theta_{\mathrm{W}}^2}. \tag{2}$$

As $\sin\theta_{\mathrm{W}}^2 \approx 1/4$ the PV asymmetry is small and highly sensitive to small changes of $\sin\theta_{\mathrm{W}}$.

Cross section and amplitudes of LO, NLO, and NNLO

Let us start by writing the cross section of polarized Møller scattering:

$$\begin{aligned} \sigma &= \frac{\pi^3}{2s}|M_0 + M_1 + M_2|^2 \approx \\ &\approx \frac{\pi^3}{2s}(M_0 M_0^+ + 2\mathrm{Re} M_1 M_0^+ + M_1 M_1^+ + 2\mathrm{Re} M_2 M_0^+) = \\ &= \qquad \text{LO} \quad + \quad \text{NLO} \quad + \quad \text{Q-part} \quad + \quad \text{T-part.} \end{aligned} \tag{3}$$

where amplitudes $M_{0,1,2}$ are proportional to powers of fine structure constant $\alpha = e^2/4\pi$:

- M_0 is the Born amplitude $\sim \mathcal{O}(\alpha)$, LO,
- M_1 is the one-loop amplitude $\sim \mathcal{O}(\alpha^2)$, NLO.
- M_2 is the two-loop amplitude $\sim \mathcal{O}(\alpha^3)$, NNLO.

LO amplitudes

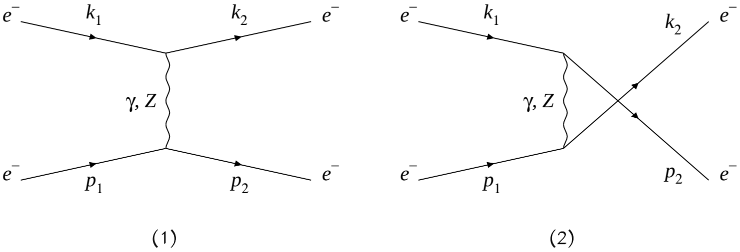

Рис. 3: LO t-channel (1) and u-channel (2) amplitudes

Mandelstam invariants:

$$s = (k_1 + p_1)^2, \ t = (k_1 - k_2)^2, \ u = (k_2 - p_1)^2.$$

Photon (γ) and Z-boson propagators:

$$D^{ir} = \frac{1}{r - m_i^2}; \quad i = \gamma, Z; \quad r = t, u.$$

LO cross section and coupling constants

Born cross section of Møller scattering can be written as:

$$\sigma^0 = \frac{\pi\alpha^2}{s} \sum_{i,j=\gamma,Z} [\lambda_-^{i,j}(u^2 D^{it} D^{jt} + t^2 D^{iu} D^{ju}) + \lambda_+^{i,j} s^2 (D^{it} + D^{iu})(D^{jt} + D^{ju})].$$

Combinations of coupling constants and the polarizations of the beam (p_B) and target (p_T) electrons:

$$\lambda_\pm^{ij} = \lambda_1{}_B^{ij} \lambda_1{}_T^{ij} \pm \lambda_2{}_B^{ij} \lambda_2{}_T^{ij},$$

$$\lambda_1{}_{B(T)}^{ij} = \lambda_V^{ij} - p_{B(T)} \lambda_A^{ij}, \quad \lambda_2{}_{B(T)}^{ij} = \lambda_A^{ij} - p_{B(T)} \lambda_V^{ij},$$

$$\lambda_V^{ij} = v^i v^j + a^i a^j, \quad \lambda_A^{ij} = v^i a^j + a^i v^j,$$

where

$$v^\gamma = 1, \ a^\gamma = 0,$$

$$v^Z = (I_e^3 + 2s_W^2)/(2s_W c_W), \ a^Z = I_e^3/(2s_W c_W), \ I_e^3 = -1/2 \ .$$

$$s_W \equiv \sin\theta_W, \ c_W \equiv \cos\theta_W = m_W/m_Z.$$

RC for polarized Møller scattering

paper	energy	renor.	details	soft	hard
Czarnecki,Marciano, PRD'96	E-158	$\overline{MS}$	NLO	No	No
Denner, Pozzorini, EPJ'99	ILC	OMS	NLO	Yes	No
Petriello, PRD'02	E-158	OMS	NLO	Yes	Yes
Zykunov, YaF'04,09	E-158, ILC	OMS	NLO	Yes	Yes
Kolomensky *et al.*, IJMP'05	E-158	OMS	NLO	Yes	Yes
our group: PRD'10	MOLLER	OMS	NLO	Yes	No
PPN'13	MOLLER	D/H RC	NLO	Yes	No
PRD'12	MOLLER	OMS	Q-part	Yes	No
EPJ'12,YaF'13,PPN'15	MOLLER	OMS	T-part	Yes	No
our group, **this report**	MOLLER	OMS	NLO	Yes	**Yes**

$\overline{MS}$ – **modified Minimal Subtraction** renormalization scheme
OMS – renormalization **On Mass Shell**
D/H RC – Denner **Renormalization Conditions** vs. Hollik RCs

NLO amplitudes (V-contribution)

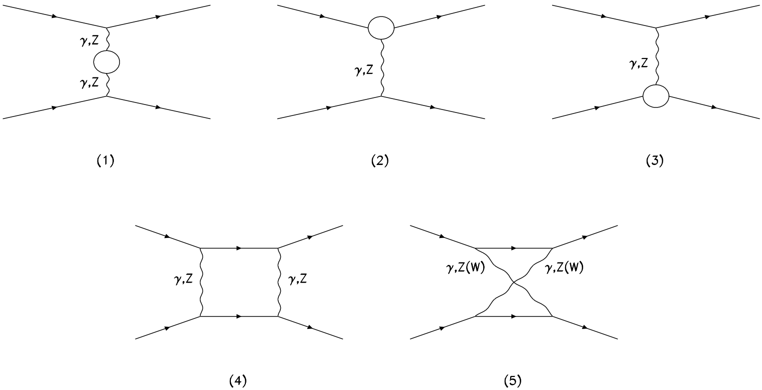

Рис. 4: The virtual t-channel one-loop diagrams: (1) – boson self-energies (BSE), (2, 3) – vertex functions (Ver), and (4, 5) – boxes. In OMS scheme no self energy of electrons

NLO amplitudes (R-contribution)

To complete the NLO radiative corrections (and to get **infrared finite** result) we need to include the **real bremsstrahlung** diagrams.

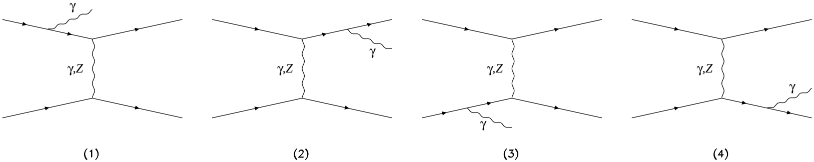

Рис. 5: t-channel diagrams for real bremsstrahlung $e^-e^- \to e^-e^-\gamma$.

Bremsstrahlung cross section

The total bremsstrahlung cross section looks like

$$\sigma^R = \frac{\alpha^3}{8s} \iiint dv\ dz\ dy \cdot \frac{1}{\pi\sqrt{R}} \frac{s-v}{s} \theta_R \sum_{i,j=\gamma,Z} M_R^{ij}, \qquad (4)$$

where for the calculation of squared matrix elements M_R^{ij} the standard Feynman rules were used.

Factor under integral sigh is

$$\theta_R = \theta(R) \cdot \theta_{IRD} \cdot \theta_{energy} \cdot \theta_{angle}, \qquad (5)$$

where energy and scattering angle of detected electrons (in lab. system) should correspond to MOLLER detector condition.

Table of Lorentz invariants

Invariant	Nonradiative	Radiative
$s = (k_1 + p_1)^2$	$2mE$	$2mE$
$t = (k_1 - k_2)^2$	$-s(1-c)/2$	$(v-s)(1-c)/2$
$u = (p_1 - k_2)^2$	$-s(1+c)/2$	$(v-s)(1+c)/2$
$v = 2kp_2$	0	v
$z = 2kk_2$	0	z
$y \equiv v_1 = 2kp_1$	0	y
$z_1 = 2kk_1$	0	$v + z - y$
$s_1 = (k_2 + p_2)^2$	s	$s - v - z$
$t_1 = (p_1 - p_2)^2$	t	$t - v + y$
$u_1 = (k_1 - p_2)^2$	u	$u + z - y$

Here k is a 4-momentum of the radiated photon.

Full bremshtrahlung photon region in 3D

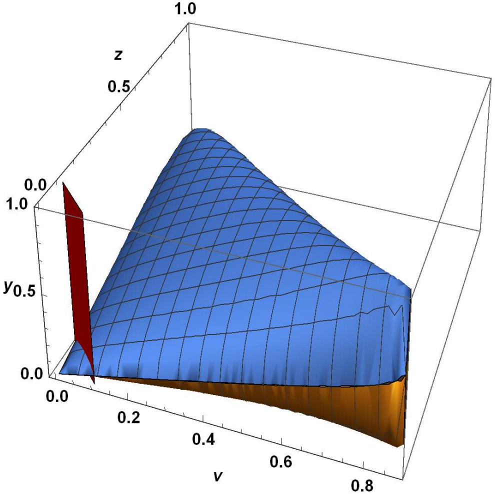

Рис. 6: Phase space of photon: red plane cuts **the infrared region**.

Detector possibility

Energies of detected electrons in Lab. system at MOLLER

$$1.8 \text{ GeV} \le E'_L \le 8.8 \text{ GeV}.$$

Interval of scattering angles in Lab. system at MOLLER is

$$5 \text{ mrad} \le \theta_L \le 19 \text{ mrad},$$

in CM system:

$$46^\circ \le \theta \le 127^\circ.$$

Hard photon region with **cuts on energies** in 3D

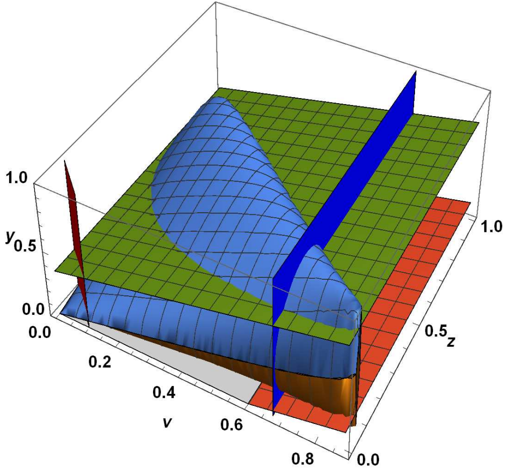

Рис. 7: Phase space of hard photon: blue plane – **1st electron**, green and orange planes – **2nd electron**.

Hard photon region with **cuts on angles** in 3D

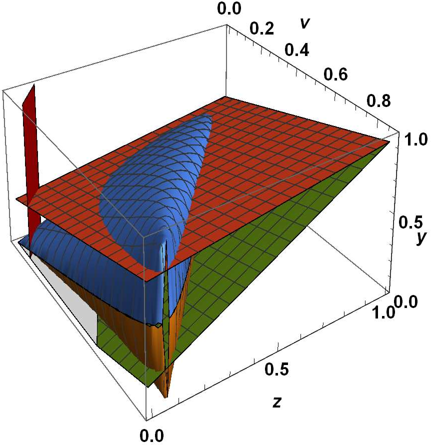

Рис. 8: Phase space of hard photon: both planes for **2nd electron**.

And all cuts together

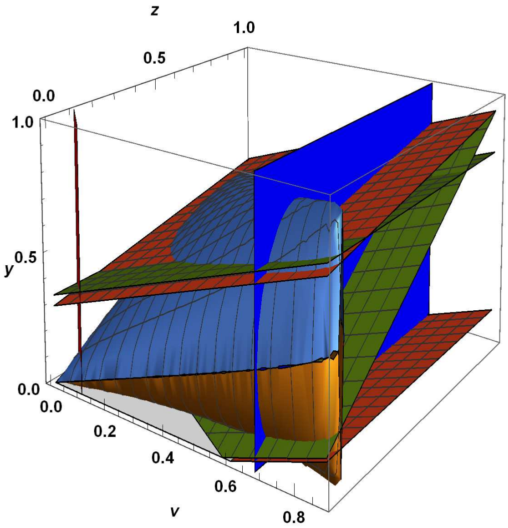

Рис. 9: Phase space of hard photon with all cuts.

How to reach the accuracy

To get a good convergence of integration in (4) (**without any approximations!)** we were forced to extremely simplify the form of result.

Key trick is to find 20 factorizing combinations of coupling constants and propagators:

$$x_k^{\pm} = \sum_{i,j=\gamma,Z} \lambda_{\pm}^{i,j} D_k^{ij}, \;\; k = \overline{1,10},$$

where

$$\begin{aligned}
D_1^{ij} &= D^{it_1} D^{jt_1}, & D_2^{ij} &= D^{it_1} D^{jt} + D^{it} D^{jt_1}, \\
D_3^{ij} &= D^{it} D^{jt}, & D_4^{ij} &= D^{it_1} D^{ju} + D^{iu} D^{jt_1}, \\
D_5^{ij} &= D^{it_1} D^{ju_1} + D^{iu_1} D^{jt_1}, & D_6^{ij} &= D^{it} D^{ju} + D^{iu} D^{jt}, \\
D_7^{ij} &= D^{it} D^{ju_1} + D^{iu_1} D^{jt}, & D_8^{ij} &= D^{iu} D^{ju}, \\
D_9^{ij} &= D^{iu} D^{ju_1} + D^{iu_1} D^{ju}, & D_{10}^{ij} &= D^{iu_1} D^{ju_1}.
\end{aligned}$$

2nd trick

is to calculate not L-part and R-part separately, **but at once the difference** $L - R$, then

$$\lambda_{+}^{i,j}\big|_{L-R} = 8\lambda_{V}^{i,j}\lambda_{A}^{i,j}, \quad \lambda_{-}^{i,j}\big|_{L-R} = 0.$$

Besides, it is easy to get unpolarized result (sum $L + R$) **by zeroing** p_B **and** p_T.

But question is how we can **construct the physical effect** from pure sum $L + R$ and difference $L - R$?

3rd trick. Using of relative corrections

The physical effect on **Born asymmetry** induced by contribution C ($C = V, R, V + R$) is defined as follows:

$$\delta_A^C = \frac{A_1^C - A_1^0}{A_1^0} = \frac{\frac{(\sigma^0+\sigma^C)|_{L-R}}{\sigma_{00}^0+\sigma_{00}^C} - \frac{\sigma^0|_{L-R}}{\sigma_{00}^0}}{\frac{\sigma^0|_{L-R}}{\sigma_{00}^0}} = \frac{\delta_-^C - \delta_+^C}{1+\delta_+^C}, \qquad (6)$$

where we used simple **relative corrections**

$$\delta_+^C = \frac{\sigma_{LL}^C + \sigma_{LR}^C + \sigma_{RL}^C + \sigma_{RR}^C}{\sigma_{LL}^0 + \sigma_{LR}^0 + \sigma_{RL}^0 + \sigma_{RR}^0} = \frac{\sigma_{00}^C}{\sigma_{00}^0}, \quad \delta_-^C = \frac{\sigma_{LL}^C - \sigma_{RR}^C}{\sigma_{LL}^0 - \sigma_{RR}^0}. \qquad (7)$$

Independance off ω

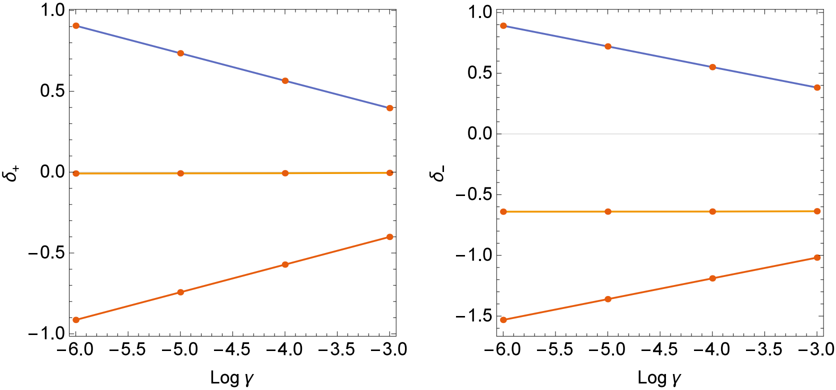

Рис. 10: Bottom line – virtual contribution, upper line – bremstrahlung, their sum is inependant off ω. We use $\gamma = \omega/\sqrt{s}$. Scattering angle $\theta = 60°$.

Relative corrections vs. angle

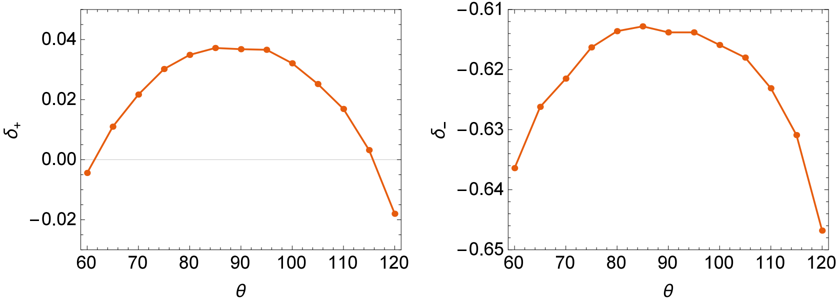

Рис. 11: Dependence of **total relative corrections** on scattering angle (in CM system) for MOLLER detector.

Physical effect

It is time to get the physical effect on **Born asymmetry**.

For example, at $\theta = 90^\circ$ and all MOLLER detector cuts:

$$\delta_A = \frac{\delta_- - \delta_+}{1 + \delta_+} = \frac{-0.6138 - 0.0368}{1 + 0.0368} = -0.6276.$$

Comparison with soft photon approach

We choose the effective ω_{eff} to match with the same statistics of experiment (i.e. fitting to the same correction δ_+ at fixed angle).

$\theta,^\circ$	δ_+	$\delta_A(hard)$	$\delta_A(soft)$	$\omega_{\text{eff}}/\sqrt{s}$
60	-0.0044	-0.6348	-0.6205	0.2070
70	0.0217	-0.6295	-0.6050	0.2932
80	0.0349	-0.6266	-0.5974	0.3480
90	0.0368	-0.6276	-0.5964	0.3565
100	0.0321	-0.6278	-0.5991	0.3355
110	0.0169	-0.6293	-0.6079	0.2753
120	-0.0180	-0.6403	-0.6292	0.1722

One happy possibility

Now we can estimate $\delta_+^{2-loop,LL}$ – two-loop leading logarithm relative correction

$$\delta_+^{2-loop,LL}(\omega) = 2\Big(\frac{\alpha}{\pi}\Big)^2 \log^2\frac{s}{m^2}\Big(2\log\frac{2\,\omega}{\sqrt{s}} + \frac{3}{2}\Big)^2 - \frac{1}{2}\Big(\frac{\alpha}{\pi}\Big)^2\frac{8}{3}\pi^2\big(\log\frac{tu}{m^2 s} - 1\big)^2.$$

Using

$$\omega \longrightarrow \omega_{\rm eff}$$

we can estimate this correction, for example, at $\theta = 90^\circ$ for MOLLER

$$\delta_+^{2-loop,LL}(\omega_{\rm eff}) = 0.000\,833 - 0.004\,872 = -0.004\,039$$

with collinear logarithm and effective logarithm part

$$\log\frac{s}{m^2} = 10.67, \quad 2\log\frac{2\,\omega_{\rm eff}}{\sqrt{s}} = 2\log(2\cdot 0.3565) = -0.68.$$

Conclusion

- We **finalize the exact calculation of NLO radiative corrections for MOLLER** as the taking into acount of hard photons in accordance with detector.
- Now **it is challenge for our group** to take into consideration the NNLO radiative corrections. The most part of two-loop calculations:
  - Q-part (PRD'12),
  - gauge invariant set of BSE and Vertices (ЯФ'13),
  - double boxes (EPJ'12),
  - boxes with insertions (PEPAN'15)

  is already done.

Aknowledgments

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Relative corrections vs. energy (from MOLLER to ILC)

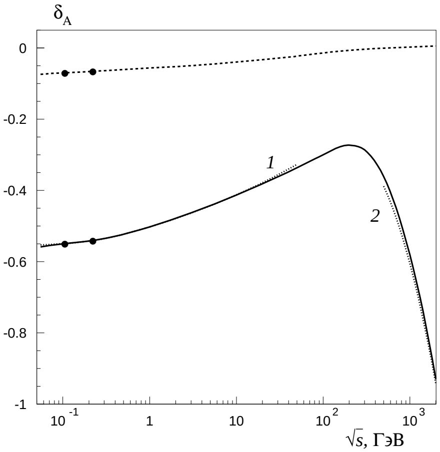

Рис. 12: Dashed line – **QED result with soft photons** at $\omega = 0.05\sqrt{s}$, solid line – exact calculation for **weak corrections**, 1 and 2 – asymptotic estimations ($\theta = 90°$; circles are points of MOLLER and E-158).