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**Form Factor of the Relativistic Two-Particle System in
the Relativistic Quasipotential Approach:
The Case of Arbitrary Masses and Vector Current**

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Plan of Talk

- 1. Introduction
- 2. Equation for the wave function
- 3. Form factor of the relativistic two-particle system
- 4. Root-mean-square radius and form factor for Coulomb interaction
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1. Introduction

The study of **hadrons electromagnetic form factors** allows to obtain the information about spatial hadrons structure. The idea of the composite quark nature of hadrons and suggestion about scale invariant behavior in the region of large momentum transfers has allowed to reveal regularity of the elastic hadrons form factors behavior [V. A. Matveev, R. M. Muradyan, A. N. Tavkhelidze, *Nuovo Cimento* **5**, 907 (1973)]. To describe the behavior of the form factors the different pole vector-dominance models (**VDM**) were used. These models successfully reproduce the behavior of the pion form factor as in space-like, so and at time-like regions [J. G. Körner, M. Kuroda, *Phys. Rev. D* **16**, 2165 (1977)], and behavior of the nucleon form factor in the space-like regions [S. Dubnicka, *Nuovo Cimento A* **103**, 1417 (1991)].

However the models **VDM** fail in description experimentally of the observed for large importances of the momentum transfer of the system $-t = Q^2$ the quick decrease of electromagnetic form factor at time-like region according to the law of dipole $\sim t^{-2}$. The reason is that the model **VDM** assume that the virtual photon flying in the nucleon "sees" only the vector mesons which there are the quark-antiquark bound-states while the structure of nucleon study at small distances where the momentum transfer of the system there is enough large value and quarks move quasifree (the asymptotic freedom).

The other approach was suggested in [O. D. Dalkarov, Pisma v ZhETF 28, 183 (1978); O. D. Dalkarov and K. V. Protasov, Nucl. Phys. A 504, 845 (1989); Mod. Phys. Lett. A 4, 1203 (1989); JETP Lett. 49, 273 (1989); O. D. Dalkarov, P. A. Khakhulin and A. Yu. Voronin, E-Print arXiv: 0906.0266v1]

for description of the behavior of baryon (nucleon) form factors at time-like region close to their threshold. In this approach the baryon (nucleon) form factor at time-like region close to \underline{BB} (\underline{NN}) threshold introduces as a product of a factor corresponding to

singularities of transition amplitude lying far from their threshold and a factor reflecting strong final state interaction. This last factor gives the energy dependency of form factor. However the problem of covariant description of form factor in the whole, rather than only in asymptotic region energy in the framework of relativistic quark model continues remain interesting. For this we must know the dynamics of interacting quarks more in detail, in particular, know the covariant wave functions their of relative motion.

Within the quantum field theory the covariant wave functions of relative motion can be obtained using the relativistic covariant two-particle quasipotential equations of Logunov–Tavkhelidze [A. A. Logunov, A. N. Tavkhelidze, *Nuovo Cim.* **29**, 380 (1963)] and Kadyshevsky [V. G. Kadyshevsky, *Sov. Phys. JETP* **19**, 443, 597 (1964); *Sov. Phys. Dokl.* **10**, 46 (1965); *Nucl. Phys. B* **6**, 125 (1968)]. The using of three-dimensional relativistic quasipotential (RQP) equation of Logunov–Tavkhelidze for description of the form factors of composite systems was executed in [V. A. Matveev, R. M. Muradyan, A. N. Tavkhelidze, *Preprint No. E2-3498, JINR (Dubna, 1967); No. P2-3900, JINR (Dubna, 1968); V. R. Garsevanishvili, A. N. Kvinikhidze, V. A. Matveev, A. N. Tavkhelidze, and R. N. Faustov, *Theor. Math. Phys.* **23**, 533 (1975); A. L. Khelashvili, *Preprint No. P2-8750, JINR (Dubna, 1976); R. N. Faustov, *Theor. Math. Phys.* **3**, 478 (1970); *Ann. Phys.* **78**, 176 (1973)].**

However, use of the [equation Logunov-Tavkhelidze](#) for wave function in the momentum representation has not allowed to research the behavior of form factor in broad interval of importances of momentum transfer of relativistic two-particle bound system. The other model of account of contribution small distances in form factor of the proton was considered in [[S. I. Bilenkaya, N. B. Skachkov, I. L. Solovtsov, Yad. Fiz. 26, 1051 \(1977\)](#)]. This model is based on invariant description of the structure of particles in [relativistic configurational space](#) that was carried in [[V. G. Kadyshevsky, R. M. Mir-Kasimov, and N. B. Skachkov, Nuovo Cimento A 55, 233 \(1968\)](#)] in the case of interaction between two relativistic spinless particles that have equal masses m in which the [Compton wavelength](#) of particle plays role of natural scale. In this model is taken into account both the contribution to the proton form factor of [vector mesons](#) and the contribution from its the [central part](#) having radius of the [Compton wavelength](#).

The method of transition to the relativistic configurational representation in the case of interaction between two relativistic spinless particles with equal masses was used in [N. B. Skachkov, I. L. Solovtsov, Preprint No. E2-11678, JINR (Dubna, 1978); Teor. Mat. Fiz. **41**, 205 (1979)] to construct the three-dimensional covariant formalism for the description of relativistic two-particle systems. Within the framework of this formalism the expressions for the form factors of relativistic two-particle systems [N. B. Skachkov, I. L. Solovtsov, Preprint No. E2-11727, JINR (Dubna, 1978); Yad. Fiz. **30**, 1079 (1979); Teor. Mat. Fiz. **43**, 330 (1980)] were obtained.

The aim of this work is to obtain the expression for the elastic form factor of relativistic two-particle system in the case of vector current on the basis of covariant Hamiltonian formulation of quantum field theory [V. G. Kadyshevsky, Sov. Phys. JETP **19**, 443, 597 (1964); Sov. Phys. Dokl. **10**, 46 (1965); Nucl. Phys. B **6**, 125 (1968)], formulated in the three-dimensional relativistic configurational representation for the interaction of two relativistic spinless particles having arbitrary masses m_1, m_2 [V. G. Kadyshevsky, M. D. Mateev, and R. M. Mir-Kasimov, Yad. Fiz. **11**, 692 (1970) [Sov. J. Nucl. Phys. **11**, 388 (1970)]; V. G. Kadyshevsky, R. M. Mir-Kasimov, and N. B. Skachkov, Fiz. Elem. Chastits At. Yadra **2**, 635 (1972) [Sov. J. Part. Nucl. **2**, 69 (1972)]].

2. Equation for the wave function

In the case of interaction between two relativistic particles with arbitrary masses m_1 and m_2 , the RQP approach developed in [V. G. Kadyshevsky, M. D. Mateev, and R. M. Mir-Kasimov, *Yad. Fiz.* **11**, 692 (1970) [*Sov. J. Nucl. Phys.* **11**, 388 (1970)]; V. G. Kadyshevsky, R. M. Mir-Kasimov, and N. B. Skachkov, *Fiz. Elem. Chastits At. Yadra* **2**, 635 (1972) [*Sov. J. Part. Nucl.* **2**, 69 (1972)]], permitted introducing the concept of an effective relativistic particle whose mass is $m' = \sqrt{m_1 m_2}$ and which plays the role of a bound two-particle system, whereby one reduces the two-body problem in question to a one-body problem treated in terms of the RQP wave function $\Psi^{M\odot}(\Delta^{p',m',\lambda\odot})$ describing the effective relativistic particle and satisfying the fully covariant RQP Kadyshevsky equation in angular momentum space, the velocity

4-vector being $\lambda_{\sigma} = (\lambda_0^{\sigma}; \lambda_{\sigma}^{\sigma})^{(a)}$, that is,

$$(2\Delta_0^{p',m',\lambda_{\sigma}} - 2\Delta_0^{d',m',\lambda_{\sigma}}) \Phi_{M_{\sigma}}(\Delta_{p',m',\lambda_{\sigma}}) = \quad (1)$$

$$= \frac{2\mu}{m'(2\pi)^3} \int d\Omega \Delta_{k',m',\lambda_{\sigma}} \tilde{V}(\Delta_{p',m',\lambda_{\sigma}}, \Delta_{k',m',\lambda_{\sigma}}; \Delta_0^{q',m',\lambda_{\sigma}}) \Phi_{M_{\sigma}}(\Delta_{k',m',\lambda_{\sigma}}),$$

$\mu = m_1 m_2 / (m_1 + m_2)$ is the ordinary reduced mass of two particles

that have arbitrary masses and $d\Omega \Delta_{k',m',\lambda_{\sigma}} = m' d\Delta_{k',m',\lambda_{\sigma}} / \Delta_0^{k',m',\lambda_{\sigma}}$

is the relativistic three-dimensional volume element in Lobachevsky space, all 4-momenta now belonging to the upper sheet of the mass

hyperboloid:

$$\Delta_{02}^{k',m',\lambda_{\sigma}} - \Delta_2^{k',m',\lambda_{\sigma}} = m'^2. \quad (2)$$

This sheet, embedded in 4-dimensional momentum space, serves as a model of relativistic non-Euclidean momentum space. On the

mass-hyperboloid sheet (2), the Lorentz group is the motion group

for this space.

^{a)} We everywhere use the system of units where $\hbar = c = 1$.

Upon choosing the pure Lorentz transformation (boost) $V_{\tilde{\sigma}}^{-1}$, corresponding to the composite particle 4-velocity $\lambda_{\tilde{\sigma}}$: $V_{\tilde{\sigma}}^{-1} \tilde{\sigma} = (M_{\tilde{\sigma}}; \mathbf{0})$, the 4-velocity components $\Delta_{k', m', \lambda_{\tilde{\sigma}}}$ from the Lobachevsky space assume the form

$$\Delta_0^{k', m', \lambda_{\tilde{\sigma}}} = (V_{\tilde{\sigma}}^{-1} k')_0 = k'_0 \lambda_{\tilde{\sigma}}^0 - \mathbf{k}' \cdot \lambda_{\tilde{\sigma}} = \sqrt{m'^2 + \Delta_2^{k', m', \lambda_{\tilde{\sigma}}}}$$

$$\Delta_{k', m', \lambda_{\tilde{\sigma}}} = V_{\tilde{\sigma}}^{-1} \mathbf{k}' = \mathbf{k}' - \lambda_{\tilde{\sigma}} \left(k'_0 - \frac{\mathbf{k}' \cdot \lambda_{\tilde{\sigma}}}{1 + \lambda_{\tilde{\sigma}}^0} \right). \quad (3)$$

Equation (1) can be considered as a relativistic generalization of the Schrödinger equation in the spirit of the Lobachevsky geometry realized on upper sheet of hyperboloid (2). This equation describes the scattering on $\tilde{V}(\Delta_{k', m', \lambda_{\tilde{\sigma}}}, \Delta_{k', m', \lambda_{\tilde{\sigma}}}, \Delta_0^{k', m', \lambda_{\tilde{\sigma}}}; \Delta_0^{k', m', \lambda_{\tilde{\sigma}}})$ of the effective relativistic particle that plays the role of a two-particle system, has m' and $\Delta_{k', m', \lambda_{\tilde{\sigma}}}$ carries $\sqrt{s_b} = M_{\tilde{\sigma}} = \frac{\hbar}{m' \Delta_0^{k', m', \lambda_{\tilde{\sigma}}}}$.

In the equation (1) it is convenient to expand over the functions

$$(4) \quad \xi(\Delta_{p',m',\lambda\vartheta}, \mathbf{r}) = \left(\frac{m'}{\Delta_0^{p',m',\lambda\vartheta} - \Delta_{p',m',\lambda\vartheta} \cdot \mathbf{n}} \right)^{-1-ir/\lambda'}$$

which realize the principal series of unitary irreducible

representations of the Lorentz group, i.e. the group of motions of the Lobachevsky space momentum, realized on upper sheet of the mass hyperboloid (2). In the nonrelativistic limit

$$|\Delta_{p',m',\lambda\vartheta}| \gg 1/\lambda', r \gg \lambda' \quad \xi(\Delta_{p',m',\lambda\vartheta}, \mathbf{r}) \rightarrow \exp(i\Delta_{p',m',\lambda\vartheta} \cdot \mathbf{r}).$$

The group parameter r in (4) plays the role of the modulus of the relativistic relative coordinate \mathbf{r} ($\mathbf{r} = r\mathbf{n}$, $|\mathbf{n}| = 1$), and $\lambda' = 1/m'$ is the Compton wavelength. This parameter enumerates the

eigenvalues of the Casimir operator of the Lorentz group

$$\hat{C}_L = (1/4)M^{\mu\nu}M^{\mu\nu} \quad (M^{\mu\nu} = p^\mu\partial/\partial p^\nu - p^\nu\partial/\partial p^\mu \text{ are the generators of the Lorentz group})$$

$$\hat{C}_L \xi(\Delta_{p',m',\lambda\vartheta}, \mathbf{r}) = (m'^{-2} + r^2)\xi(\Delta_{p',m',\lambda\vartheta}, \mathbf{r}) \text{ is a relativistic invariant } (0 \leq r < \infty).$$

The functions in (4) obey the conditions

$$\int \frac{1}{\Omega} d\Omega \Delta_{\theta, \varphi}^{p', m', d} \xi_{*}(\mathbf{r}, \theta, \varphi) \Delta_{\theta, \varphi}^{p', m', d} \xi_{*}(\mathbf{r}', \theta, \varphi) = \delta(\mathbf{r}' - \mathbf{r}),$$

$$\int \frac{1}{\Omega} d\Omega \xi_{*}(\mathbf{r}, \theta, \varphi) \Delta_{\theta, \varphi}^{p', m', d} \xi_{*}(\mathbf{r}', \theta, \varphi) \Delta_{\theta, \varphi}^{p', m', d} =$$

$$(5) \quad \Delta_{\theta, \varphi}^{p', m', d} \delta(\theta, \varphi) - \Delta_{\theta, \varphi}^{p', m', d}(\theta, \varphi),$$

and they satisfy the equation in terms of finite differences

$$(6) \quad (2\Delta_{\theta, \varphi}^{p', m', d} - \widehat{H}_0) \xi(\Delta_{\theta, \varphi}^{p', m', d}, \mathbf{r}) = 0.$$

$$\widehat{H}_0 = 2m' \left[\cosh \left(\frac{\partial}{\partial r} \right) + \frac{r}{\sinh} \left(\frac{\partial}{\partial r} \right) - \frac{\lambda'^2}{2r^2} \Delta_{\theta, \varphi} \right] \exp \left(\frac{\partial}{\partial r} \right)$$

is the operator of the free Hamiltonian, while $\Delta_{\theta, \varphi}$ is its angular

part.

The wave RQP-functions in the momentum space and the \mathbf{r} -representation (the relativistic configuration representation) are related

$$\psi_{M\varnothing}(\mathbf{r}) = \frac{1}{\varepsilon} \int d^3p \Delta_{\varnothing}^{\lambda', m', d} \xi_{\varnothing}(\Delta_{\varnothing}^{\lambda', m', d}, \mathbf{r}) \Psi_{M\varnothing}(\Delta_{\varnothing}^{\lambda', m', d}),$$

$$\Psi_{M\varnothing}(\Delta_{\varnothing}^{\lambda', m', d}) = \int d^3r \xi_{\varnothing}^*(\Delta_{\varnothing}^{\lambda', m', d}, \mathbf{r}) \psi_{M\varnothing}(\mathbf{r}). \quad (7)$$

For the local quasipotential

$$\begin{aligned} \tilde{V} &\equiv \tilde{V}(\Delta_{\varnothing}^{\lambda', m', d}, \Delta_{\varnothing}^{\lambda', m', k'}, \Delta_{\varnothing}^{\lambda', m', b}, \Delta_{\varnothing}^{\lambda', m', \lambda\varnothing}) \\ &\equiv \tilde{V}(\Delta_{\varnothing}^{\lambda', m', d}, \Delta_{\varnothing}^{\lambda', m', k'}, \Delta_{\varnothing}^{\lambda', m', b}, \Delta_{\varnothing}^{\lambda', m', \lambda\varnothing}) \end{aligned} \quad (8)$$

square of the vector of momentum transfer in the Lobachevsky space $\Delta_{\varnothing}^{\lambda', k'} = \mathbf{p}'(-)\mathbf{k}'$ is the Loretz invariant that allows to present it in the form

By using the transformations (7) eq. (1) takes the form

$$(6) \quad 2\Delta_0^b \nabla_0^b \psi_{M\sigma}(\mathbf{r}) - \widehat{H}_0 \psi_{M\sigma}(\mathbf{r}) = \frac{m'}{2\mu} V(\mathbf{r}; \Delta_0^b \nabla_0^b) \psi_{M\sigma}(\mathbf{r}),$$

$$V(\mathbf{r}; \Delta_0^b \nabla_0^b) = \frac{1}{3} \int d\Omega \Delta_0^b \nabla_0^b \xi(\Delta_0^b \nabla_0^b, \mathbf{r}) \widetilde{V}(\Delta_0^b \nabla_0^b, \mathbf{r}; \Delta_0^b \nabla_0^b).$$

For spherically symmetric potentials

$$(10) \quad \psi_{M\sigma}(\mathbf{r}) = \sum_{\ell=0}^{\infty} (2\ell+1) \varphi_{\ell}(r, \chi) P_{\ell} \left(\frac{\Delta_0^b \nabla_0^b |r|}{\Delta_0^b \nabla_0^b |r|} \right),$$

we obtain equation for the partial wave function in the form

$$\left[\cosh \left(\chi' \frac{d}{d} \right) + \frac{\chi'^2 (\ell+1)}{2r(r+\chi')} \exp \left(\chi' \frac{d}{d} \right) - X(r) \right] \varphi_{\ell}(r, \chi) = 0,$$

$$X(r) = \frac{\mu}{r^2} (M_{\sigma} - V(r; \chi)),$$

and χ is the rapidity: $\Delta_0^b \nabla_0^b \sinh \chi \Delta_0^b \nabla_0^b = m' \sinh \chi \Delta_0^b \nabla_0^b$, $|\Delta_0^b \nabla_0^b| = 1$, $M_{\sigma} = m'/\mu (\Delta_0^b \nabla_0^b)^2 = m' \cosh \chi$.

3. Form factor of the relativistic two-particle system

For simplicity we consider here only the case of spinless field when the Hamiltonian density is given by the expression

$$H(x) = -z_1 \varphi_+^1(x) \varphi_1(x) A(x) - z_2 \varphi_+^2(x) \varphi_2(x) A(x). \quad (11)$$

In ref. [N. B. Skachkov, I. L. Solovtsov, Preprint No. E2-11727, JINR (Dubna, 1978); Yad. Fiz. **30**, 1079 (1979)], founded on refs. [V. A.

Matveev, R. M. Muradyan, A. N. Tavkhelidze, Preprint No. E2-3498, JINR (Dubna, 1967); No. P2-3900, JINR (Dubna, 1968); V. R. Garsevanshvilii, A. N. Kvinikhidze, V. A. Matveev, A. N. Tavkhelidze, and R. N. Faustov, Theor. Math. Phys. **23**, 533 (1975); A. L. Khelashvili, Preprint No. P2-8750, JINR (Dubna, 1976); R. N. Faustov, Theor. Math. Phys. **3**, 478 (1970); Ann. Phys.

78, 176 (1973)], the form factor of two-particle system was defined as the matrix element of the local current operator between bound states with the 4-momentum P , Q through the covariant wave ROP-functions satisfying Eq. (1).

Then, as follows from refs. [N. B. Skachkov, I. L. Solovtsov, Preprint No. E2-11727, JINR (Dubna, 1978); Yad. Fiz. **30**, 1079 (1979); Teor. Mat. Fiz. **43**, 330 (1980)], the invariant expression in the momentum representation for the matrix element of the local current operator near poles of bound states with the 4-momentum P, Q for the interaction of two relativistic spinless particles with arbitrary masses m_1, m_2 has the form

$$\begin{aligned}
 & \langle P | J_\nu | Q \rangle = \frac{z_1}{(2\pi)^3} \int d^4p d^4q d^4k_1 d^4k_2 d^4k'_1 \theta(k_{20}) \delta(k_2^2 - m_2^2) \times \\
 & \times \Gamma_P^+(k'_1, k_2; \lambda_{PTP}) \frac{(k_1 + k'_1)_\nu}{(TP + i\varepsilon)(TQ - i\varepsilon)} \Gamma_Q(k_1, k_2; \lambda_{QTQ}) \times \\
 & \times \theta(k_{10}) \delta(k_1^2 - m_1^2) \theta(k'_{10}) \delta(k'_{12} - m_1^2) \delta^{(4)}(Q + k_1 + k_2 + \lambda_{QTQ}) \times \\
 & \times \delta^{(4)}(P - k_2 - k'_1 - \lambda_{PTP}) + (1 \leftrightarrow 2),
 \end{aligned}
 \tag{12}$$

$$\lambda_P = P/\sqrt{P^2}, \lambda_Q = Q/\sqrt{Q^2}, k_2^i = k_{20}^i - k_2^i = m_2^i, i = 1, 2.$$

This equation answers the diagram on fig. 1.

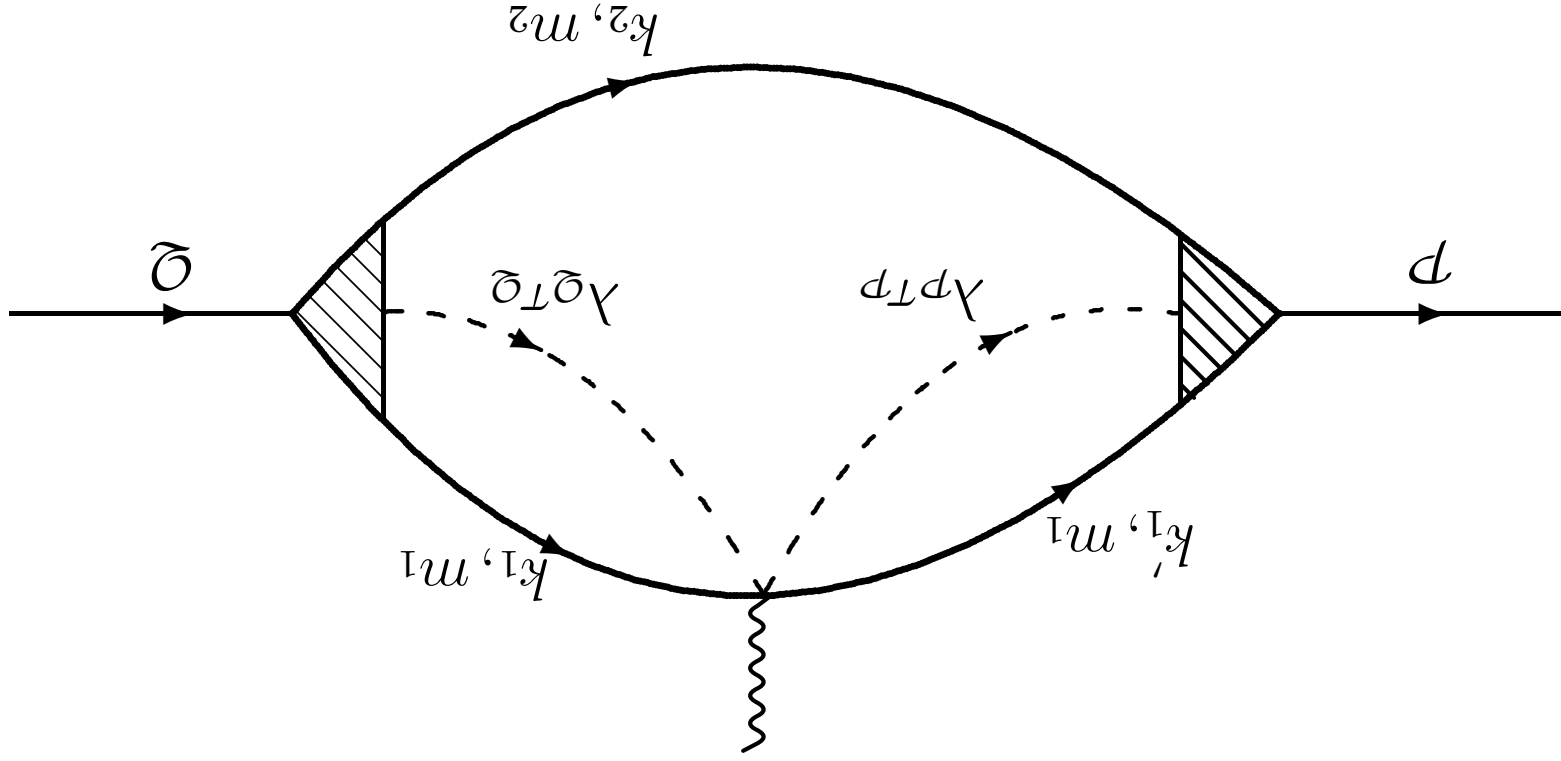


Fig. 1: The diagram for the matrix element of the local current operator between bound states with the 4-momentum P, Q for the interaction of two relativistic spinless particles with arbitrary masses.

Here follows to emphasize that because of transition to different own timeses of the system before ($\tau_{\tilde{Q}} = \lambda_{\tilde{Q}} X, X = x_1 + x_2$) and after interaction ($\tau_P = \lambda_P X$) the diagram on fig. 1 differ from diagrams, which appear in approach of the Kadyshevsky for *S-matrix*. The 4-velocities of the composite particle before, $\lambda_{\tilde{Q}}$, and after interaction, λ_P , will differ also.

In the case of equal quark masses ($m_1 = m_2 = m$) and for real-valued wave functions, expression (12) for the matrix element of the vector-current operator satisfies the transverseness condition

$$(P - \tilde{Q})^\nu \langle P | J_\nu | \tilde{Q} \rangle = 0, \quad (13)$$

In the case of unequal quark masses ($m_1 \neq m_2$), expression (12) features additionally its transverse component, which breaks the transverseness condition in (13).

Therefore, the 4-vector in expression (12) can be represented in the form

$$(14) \quad \langle P|J_\nu|\tilde{Q}\rangle = F^{(+)}(t)(P + \tilde{Q})_\nu + F^{(-)}(t)(P - \tilde{Q})_\nu.$$

Thus, it is necessary to multiply expression (12) by $(P \pm \tilde{Q})^\nu$ and to consider that, at $\tilde{Q}^2 = M_{\tilde{Q}}^2$, $P^2 = M_P^2$, the following relation holds:

$$t = (P - \tilde{Q})^2 = -Q^2 = M_{\tilde{Q}}^2 + M_P^2 - 2P\tilde{Q}, \quad 2P\tilde{Q} = M_{\tilde{Q}}^2 + M_P^2 - t,$$

$$(15) \quad (P + \tilde{Q})^2 = M_{\tilde{Q}}^2 + M_P^2 + 2P\tilde{Q} = 2(M_{\tilde{Q}}^2 + M_P^2) - t.$$

Performing integration with respect to $dk_{20}, dk_{10}, dk'_{10}$ and taking into account Eqs. (14) and (15), we obtain the following expressions for the form-factor components:

$$F^{(+)}(t) = \frac{(2M_2^{\tilde{Q}} + 2M_2^P - t)(4\pi)^3}{z_1} \int \frac{d\tau p d\tilde{q} dk_2 dk_1 dk_1'}{\sqrt{m_2^2 + k_2^2} \sqrt{m_1^2 + k_1^2} \sqrt{m_2^2 + k_1'^2}} \times$$

$$\times \Gamma_+^P(k_1', k_2; \lambda_{P\tau P}) \frac{(P + \tilde{Q})(k_1 + k_1')}{(TP + i\varepsilon)(T\tilde{Q} - i\varepsilon)} \Gamma_{\tilde{Q}}(k_1, k_2; \lambda_{\tilde{Q}T\tilde{Q}}) \times$$

$$\times \delta^{(4)} \left[-1 + \frac{M_{\tilde{Q}}}{T\tilde{Q}} \tilde{Q} + k_1 + k_2 \right] \delta^{(4)} \left[1 - \frac{M_P}{TP} P - k_2 - k_1' \right] + (1 \leftrightarrow 2),$$

$$(16)$$

$$F^{(-)}(t) = \frac{z_1}{it(4\pi)^3} \int \frac{d\tau p d\tilde{q} dk_2 dk_1 dk_1'}{\sqrt{m_2^2 + k_2^2} \sqrt{m_1^2 + k_1^2} \sqrt{m_2^2 + k_1'^2}} \times$$

$$\times \Gamma_+^P(k_1', k_2; \lambda_{P\tau P}) \frac{(P - \tilde{Q})(k_1 + k_1')}{(TP + i\varepsilon)(T\tilde{Q} - i\varepsilon)} \Gamma_{\tilde{Q}}(k_1, k_2; \lambda_{\tilde{Q}T\tilde{Q}}) \times$$

$$\times \delta^{(4)} \left[-1 + \frac{M_{\tilde{Q}}}{T\tilde{Q}} \tilde{Q} + k_1 + k_2 \right] \delta^{(4)} \left[1 - \frac{M_P}{TP} P - k_2 - k_1' \right] + (1 \leftrightarrow 2).$$

$$(17)$$

Within this approach, for the bounded system of spinless particles which are found in the motion with moment $J = 0$ the vertex

functions $\Gamma_{\tilde{Q}}(k_2, k_1; \lambda_{\tilde{Q}TP})$ and $\Gamma_P(k_2, k_1; \lambda_{PTP})$ when $\lambda_{\tilde{Q}} \parallel \tilde{Q}$

and $\lambda_P \parallel P$ will depend each only on one the Lorentz invariant scalar parameter, as which we choose accordingly $\tilde{Q}k_2$ and Pk_2 .

According to Eqs. (3), these parameters are invariant under the pure Lorentz transformations: $\lambda_{\tilde{Q}}^{-1} \tilde{Q} = (M_{\tilde{Q}}; \mathbf{0}), \lambda_P^{-1} P = (M_P; \mathbf{0})$;

$$\tilde{Q}k_2 = \lambda_{\tilde{Q}}^{-1}(\tilde{Q}k_2) = (\lambda_{\tilde{Q}}^{-1} \tilde{Q})(\lambda_{\tilde{Q}}^{-1} k_2) = M_{\tilde{Q}} \Delta_0^{k_2, m_2 \lambda_{\tilde{Q}}} = M_{\tilde{Q}} \sqrt{m_2^2 + \Delta_2^{k_2, m_2 \lambda_{\tilde{Q}}}},$$

$$Pk_2 = \lambda_P^{-1}(Pk_2) = M_P \Delta_0^{k_2, m_2 \lambda_P} = M_P \sqrt{m_2^2 + \Delta_2^{k_2, m_2 \lambda_P}}.$$

Moreover, the application of the Lorentz transformation to the conservation laws

$$-\tilde{Q} + k_1 + k_2 + \lambda_{\tilde{Q}T\tilde{Q}} = 0, P - k_2 - k_1' - \lambda_{PTP} = 0, \tag{18}$$

yields

$$\begin{aligned}
 T_{\mathcal{Q}} = M_{\mathcal{Q}} - \Delta_0^{k_2, m_2 \lambda_{\mathcal{Q}}} - \Delta_0^{k_1, m_1 \lambda_{\mathcal{Q}}}, \quad \Delta_{k_1, m_1 \lambda_{\mathcal{Q}}}, \Delta_{k_2, m_2 \lambda_{\mathcal{Q}}}; \\
 T_{\mathcal{P}} = M_{\mathcal{P}} - \Delta_0^{k_2, m_2 \lambda_{\mathcal{P}}} - \Delta_0^{k'_1, m_1 \lambda_{\mathcal{P}}}, \quad \Delta_{k'_1, m_1 \lambda_{\mathcal{P}}}, \Delta_{k_2, m_2 \lambda_{\mathcal{P}}}.
 \end{aligned}
 \tag{19}$$

From Eqs. (18) and (19), it also follows that

$$k_1 + k'_1 = \lambda_{\mathcal{Q}} \sqrt{s_{\Delta_{k_2, m_2 \lambda_{\mathcal{Q}}}}} + \lambda_{\mathcal{P}} \sqrt{s_{\Delta_{k_2, m_2 \lambda_{\mathcal{P}}}}} - 2k_2,$$

where we have used the invariance of the total energy

$$\begin{aligned}
 \sqrt{s_{\mathcal{K}}} = \sqrt{(k_2 + k_1)^2} = \sqrt{s_{\Delta_{k_2, m_2 \lambda_{\mathcal{Q}}}}} = \sqrt{m_2^2 + \Delta_2^{k_2, m_2 \lambda_{\mathcal{Q}}}} + \\
 \sqrt{s_{\mathcal{K}'}} = \sqrt{(k_2 + k'_1)^2} = \sqrt{s_{\Delta_{k_2, m_2 \lambda_{\mathcal{P}}}}} = \sqrt{m_2^2 + \Delta_2^{k_2, m_2 \lambda_{\mathcal{P}}}} + \\
 \sqrt{m_2^2 + \Delta_2^{k'_1, m_1 \lambda_{\mathcal{P}}}}.
 \end{aligned}
 \tag{20}$$

Taking into account Eq. (15), we find from here that

$$(P \mp Q)(k_1 + k'_1) = \frac{M_Q^2 + M_P^2 - t}{2M_Q} \sqrt{s_{\Delta k_2, m_2 \lambda_Q}} \mp \quad (21)$$

$$\mp \frac{2M_P}{M_Q^2 + M_P^2 - t} \sqrt{s_{\Delta k_2, m_2 \lambda_P}} \mp \frac{(m_1^2 - m_2^2)M_Q}{(m_1^2 - m_2^2)(M_P)} \sqrt{s_{\Delta k_2, m_2 \lambda_P}} + \frac{(m_1^2 - m_2^2)M_Q}{(m_1^2 - m_2^2)(M_P)} \sqrt{s_{\Delta k_2, m_2 \lambda_Q}}.$$

In order to carry out integration in expressions (16) and (17) with respect to dk_1, dk'_1, dT_Q, dT_P , we perform the pure Lorentz transformations $V_{-1}^{\lambda_Q}$ and $V_{-1}^{\lambda_P}$ by formulas (3) in the integrals with respect to dk_1 and dk'_1 , correspondingly, and take into account Eq. (21) and the invariance of the delta functions involved and the integration measures d^4k_i to (16) and (17) on the mass hyperboloids (12) under Lorentz transformations.

As a result expressions (16) and (17) takes the form:

$$\begin{aligned}
 F^{(+)}(t) = & \frac{z_1}{(2M_2^{\bar{Q}} + 2M_P^2 - t)(4\pi)^3} \int \frac{d\Delta_{k_2, m_2 \lambda_{\bar{Q}}}}{\sqrt{m_2^2 + \Delta_2^2 + \Delta_2}} \times \\
 & \times \frac{\Gamma_{M_P}^+ (\Delta_{k_2, m_2 \lambda_P})}{\left[M_2^{\bar{Q}} + M_P^2 - t \right]} \left[\frac{\sqrt{m_2^2 + \Delta_2^2 + \Delta_2} (M_P - \sqrt{s_{\Delta_{k_2, m_2 \lambda_P}}}) + i\epsilon \right]}{2M_{\bar{Q}}} \sqrt{s_{\Delta_{k_2, m_2 \lambda_{\bar{Q}}}}} + \\
 & + \frac{M_2^{\bar{Q}} + M_P^2 - t}{2M_P} \sqrt{s_{\Delta_{k_2, m_2 \lambda_P}}} + (m_2^2 - m_1^2) \left(\frac{M_{\bar{Q}}}{\sqrt{s_{\Delta_{k_2, m_2 \lambda_{\bar{Q}}}}} + \right. \\
 & \left. + \frac{M_P}{\sqrt{s_{\Delta_{k_2, m_2 \lambda_P}}}} \right) \Gamma_{M_{\bar{Q}}} (\Delta_{k_2, m_2 \lambda_{\bar{Q}}}) \left(M_{\bar{Q}} - \sqrt{s_{\Delta_{k_2, m_2 \lambda_{\bar{Q}}}}} - i\epsilon \right) + (1 \leftrightarrow 2),
 \end{aligned}
 \tag{22}$$

$$F^{(-)}(t) = \frac{z_1}{3} \int \frac{d\Delta_{k_2, m_2 \lambda_{\bar{Q}}}}{\sqrt{m_2^2 + \Delta_2^2}} \times \quad (23)$$

$$\times \frac{\Gamma_{M_P}^+ (\Delta_{k_2, m_2 \lambda_P}) \left[\frac{\sqrt{m_2^2 + \Delta_2^2}}{M_P} - \sqrt{s_{\Delta_{k_2, m_2 \lambda_P}}} + i\epsilon \right]}{M_2^{\bar{Q}} + M_P^2 - t} \frac{2M_{\bar{Q}}}{\sqrt{s_{\Delta_{k_2, m_2 \lambda_{\bar{Q}}}}}}$$

$$- \frac{M_2^{\bar{Q}} + M_P^2 - t}{2M_P} \frac{\sqrt{s_{\Delta_{k_2, m_2 \lambda_P}}}}{(m_2^1 - m_2^2)} \left(\frac{M_{\bar{Q}}}{\sqrt{s_{\Delta_{k_2, m_2 \lambda_{\bar{Q}}}}}} - \frac{M_P}{\sqrt{s_{\Delta_{k_2, m_2 \lambda_P}}}} \right)$$

$$\left. \frac{M_P}{\sqrt{s_{\Delta_{k_2, m_2 \lambda_P}}}} \right) \frac{\Gamma_{M_{\bar{Q}}} (\Delta_{k_2, m_2 \lambda_{\bar{Q}}})}{\sqrt{m_2^1 + \Delta_2^2}} \left(M_{\bar{Q}} - \sqrt{s_{\Delta_{k_2, m_2 \lambda_{\bar{Q}}}}} - i\epsilon \right) + (1 \leftrightarrow 2),$$

where we have introduced the notation

$$\Gamma_{\bar{Q}}(k_1, k_2; \lambda_{\bar{Q}T\bar{Q}}) = \Gamma_{M_{\bar{Q}}} (\Delta_{k_2, m_2 \lambda_{\bar{Q}}}),$$

$$\Gamma_P(k_1, k_2; \lambda_{PTP}) = \Gamma_{M_P} (\Delta_{k_2, m_2 \lambda_P}).$$

Within the RQP approach being considered, the twobody problem under study reduces to a one-body problem formulated in terms of the RQP wave function $\Psi^{M_{\mathcal{G}}}(\Delta_{k',m'}\lambda_{\mathcal{G}})$ describing an effective relativistic particle and satisfying the fully covariant RQP Kadyshevsky equation (1) in the angular-momentum space. The 4-vector k' follows to choose in the form

$$k' = (k'_0; \mathbf{k}') = \sqrt{\frac{\mathcal{K}_2}{\mathcal{K}_2^{\perp}}} \mathcal{K}_{\perp}, \quad \mathcal{K} = \frac{m_1 k_2 - m_2 k_1}{m_1 + m_2}, \quad (24)$$

the vector $\mathcal{K}_{\perp} = \mathcal{K} - \lambda_{\mathcal{K}}(\lambda_{\mathcal{K}}\mathcal{K})$ is the Wightman-Gårding vector, and $\lambda_{\mathcal{K}} = (k_1 + k_2)/\sqrt{s_k} = \lambda_{\mathcal{G}}$. Thence follows

$$\mathcal{K}_{\perp} = \frac{k_2 - k_1}{2} - \frac{m_2^2 - m_1^2}{2\sqrt{s_k}} \lambda_{\mathcal{K}}, \quad (\lambda_{\mathcal{K}}\mathcal{K}_{\perp}) = 0,$$

$$k'^2 = k_0'^2 - \mathbf{k}'^2 = \mathcal{K}_2 = \frac{m_1 m_2}{(m_1 + m_2)^2} [(m_1 + m_2)^2 - s_k].$$

Under the Lorentz transformation follows that

$$\Delta_{k_1, m_1 \lambda_\sigma}^{-1} = -\Delta_{k_2, m_2 \lambda_\sigma}^{-1} \lambda_\sigma^{-1} = (1; \mathbf{0}), \quad \Delta_{k_1, m_1 \lambda_\sigma}^{-1} \mathcal{K}_\perp = (0; \Delta_{k_2, m_2 \lambda_\sigma}^{-1}),$$

$$\Delta_{k_1, m_1 \lambda_\sigma}^{-1} \mathcal{K} = \left(m_1 \sqrt{m_2^2 + \Delta_{k_2, m_2 \lambda_\sigma}^{-2}} - m_2 \sqrt{m_1^2 + \Delta_{k_2, m_2 \lambda_\sigma}^{-2}}; \Delta_{k_2, m_2 \lambda_\sigma}^{-1} \right).$$

Then for the square of relative 3-momentum $\Delta_{k', m' \lambda_\sigma}^{-1}$ of the effective relativistic particle, having mass m' , we get $(\Delta_{k_0}^{-1} k'_0 = 0)$

$$\Delta_2^{-1} k'_{m' \lambda_\sigma} = -(\Delta_{k_1}^{-1} k'_1)_2 = -(\Delta_{k_1}^{-1} \mathcal{K})_2 = \frac{m_1 m_2}{m_1 m_2} \left[s_{\Delta_{k_2, m_2 \lambda_\sigma}^{-1}} - (m_1 + m_2)_2 \right].$$

As direction of the vector $\Delta_{k', m' \lambda_\sigma}^{-1}$, we choose the direction of the vector $\Delta_{k_2, m_2 \lambda_\sigma}^{-1}$. We then obtain:

$$\begin{aligned}
 \Delta_{k', m' \lambda \sigma} &= \sqrt{\frac{(A_{-1}^{-1} \mathcal{K})_2}{(A_{-1}^{-1} \mathcal{K}_{\perp})^2} (A_{\lambda \sigma}^{-1} \mathcal{K}_{\perp})^2} = \frac{\Delta_{k_2, m_2 \lambda \sigma}}{\Delta_{k_2, m_2 \lambda \sigma}} \left[\Delta_{k_2, m_2 \lambda \sigma} \right]_{1/2} - \\
 &\quad \left(m_1 \sqrt{m_2^2 + \Delta_2} \sqrt{m_2^2 + \Delta_2} - m_2 \sqrt{m_1^2 + \Delta_2} \sqrt{m_2^2 + \Delta_2} \right)_{1/2}, \quad (25)
 \end{aligned}$$

$$\begin{aligned}
 \frac{m'}{m'} \frac{2\mu}{2\mu} f_{-1}^{\pm} (\Delta_{k', m' \lambda \sigma}) &= \frac{\sqrt{m_2^2 + \Delta_2} \sqrt{m_2^2 + \Delta_2}}{m'} f_{-1}^{\pm} (\Delta_{k', m' \lambda \sigma}), \quad (26) \\
 \frac{m'}{m'} \frac{2\mu}{2\mu} f_{-1}^{\pm} (\Delta_{k', m' \lambda \sigma}) &= \frac{\sqrt{m_2^2 + \Delta_2} \sqrt{m_2^2 + \Delta_2}}{m'} f_{-1}^{\pm} (\Delta_{k', m' \lambda \sigma}), \quad (27)
 \end{aligned}$$

Consequently, under the transformation (25), we find that

$$(28) \quad \frac{\sqrt{m_2^2 + \Delta_2^{k_2, m_2 \lambda_0}}}{1} = \frac{\sqrt{m_2^2 + \Delta_2^{k_2, m_2 \lambda_0}}}{\sqrt{m_2^2 + \Delta_2^{k_2, m_2 \lambda_0}} + \Delta_2^{k_2, m_2 \lambda_0}} = \left(\frac{m'}{2\mu}\right)_2 f_+ (\Delta_2^{k_2, m_2 \lambda_0}) f_- (\Delta_2^{k_2, m_2 \lambda_0}),$$

and we install the inverse transformation:

$$(29) \quad \Delta_2^{k_2, m_2 \lambda_0} = \frac{m'}{2\mu} \sqrt{\frac{4\mu^2 + \Delta_2^{k_2, m_2 \lambda_0}}{m_2^2 + \Delta_2^{k_2, m_2 \lambda_0}}}.$$

The relativistic three-dimensional volume element in the Lobachevsky space is converted as follows:

$$\begin{aligned}
 &= \frac{d\Delta_{k_2, m_2} \lambda_{\varnothing}}{\sqrt{m_2^2 + \Delta_{k_2, m_2}^2} \sqrt{m_1^2 + \Delta_{k_2, m_2}^2} \lambda_{\varnothing}} \\
 &= \frac{m'}{m} \frac{2\mu \sqrt{m'^2 + \Delta_{k', m'}^2} f(\Delta_{k', m'} \lambda_{\varnothing})}{d\Delta_{k', m'} \lambda_{\varnothing}} = \dots
 \end{aligned}
 \tag{30}$$

where

$$\frac{f(\Delta_{k', m'} \lambda_{\varnothing})}{\sqrt{4\mu^2 + \Delta_{k', m'}^2} \lambda_{\varnothing}} = \dots
 \tag{31}$$

Now in (22) and (23) we perform the change of variables according to Eqs. (25) and (29) and take into account expression (30). The expressions for the components of the elastic form factor $(M_p = M_{\varnothing} = M)$ then assume the form

$$F_{(-)}(t) = \frac{(z_1 + z_2)(2M^2 - t)}{2\mu} \int \mathcal{D}\Omega \Delta_{k', m', \lambda_p} \Psi_{*M}(\Delta_{k', m', \lambda_p}) \times \quad (33)$$

$$\left[f_{+}(\Delta_{k', m', \lambda_p}) + f_{-}(\Delta_{k', m', \lambda_p}) \right] \frac{2f(\Delta_{k', m', \lambda_p})}{\Delta_0^{k', m', \lambda_p} - \Delta_0^{k', m', \lambda_p}} \Psi_M(\Delta_{k', m', \lambda_p}) +$$

$$+ \frac{(z_1 - z_2)(m_1^2 - m_2^2)M}{2\mu} \left(\frac{m'}{2\mu} \right) \int \mathcal{D}\Omega \Delta_{k', m', \lambda_p} \Psi_{*M}(\Delta_{k', m', \lambda_p}) \times$$

$$\left[f_{+}(\Delta_{k', m', \lambda_p}) + f_{-}(\Delta_{k', m', \lambda_p}) \right] \frac{2f(\Delta_{k', m', \lambda_p})}{\Delta_0^{k', m', \lambda_p} - \Delta_0^{k', m', \lambda_p}} \Psi_M(\Delta_{k', m', \lambda_p}) \times$$

where we have defined the wave function for the system in the

angular-momentum space as

$$\Psi_M(\Delta_{k', m', \lambda_p}) = \frac{f(\Delta_{k', m', \lambda_p}) \Psi_M(\Delta_{k', m', \lambda_p})}{2^{3/2} \sqrt{m'} \left(\frac{2\mu M}{2\Delta_0^{k', m', \lambda_p}} - \frac{m'}{2\Delta_0^{k', m', \lambda_p}} \right)},$$

and have introduced the relation

$$\Gamma_{M\vartheta}(\Delta_{k_2, m_2 \lambda_p}) = \Gamma_M(\Delta_{k', m', \lambda_p}), \Gamma_{M^p}(\Delta_{k_2, m_2 \lambda_p}) = \Gamma_M(\Delta_{k', m', \lambda_p}).$$

The factor can be reduced to a simpler form

$$(34) \quad \frac{f_+(\Delta_{k',m'\lambda_p}) + f_-(\Delta_{k',m'\lambda_p})}{2f(\Delta_{k',m'\lambda_p})} \approx 1 + \frac{m'^2 - 4\mu^2}{2\Delta_{02}^2 \Delta_{k',m'\lambda_p}},$$

$$\frac{m' \sqrt{m'^2 - 4\mu^2}}{2\Delta_{02}^2 \Delta_{k',m'\lambda_p}} > 1, \quad \frac{2\Delta_{02}^2 \Delta_{k',m'\lambda_p}}{m'^2 - 4\mu^2} > 1.$$

Furthermore, the vector from the Lobachevsky space $\Delta_{k',m'\lambda_p}$ arising on the upper mass-hyperboloid sheet (2) can be represented

in the form

$$\Delta_{k',m'\lambda_p} = \mathbf{k}'(-) = \mathbf{k}'(-)_{m'\lambda_p} = V^{-1} \mathbf{k}' = V^{-1} \Delta_{k',m'\lambda_p} = V(\Delta_{k',m'\lambda_p}, \mathcal{P})(-) \frac{M}{m'} \Delta_{p,\mathcal{O}},$$

$\Delta_{p,\mathcal{O}} = V^{-1} \Delta_{\lambda_{\mathcal{O}}}$ is the 4-vector of momentum transfer of the system:

$$\Delta_{p, \tilde{Q}} = V^{-1} p = p - \frac{M}{\tilde{Q}} \left(p_0 - \frac{\tilde{Q}_0 + M}{p \cdot \tilde{Q}} \right) = M \sinh \chi_{\Delta} n_{\Delta},$$

$$\Delta_{p, \tilde{Q}}^0 = (V^{-1} p)_0 = \frac{M}{p_0 \tilde{Q}_0 - p \cdot \tilde{Q}} = \frac{M}{p \tilde{Q}} = M \cosh \chi_{\Delta},$$

$$p = M \sinh \chi_{p n_p}, \tilde{Q} = M \sinh \chi_{\tilde{Q} n_{\tilde{Q}}}, p_0 = M \cosh \chi_p,$$

$$\tilde{Q}_0 = M \cosh \chi_{\tilde{Q}}, |n_p| = |n_{\tilde{Q}}| = |n_{\Delta}| = 1, \Delta_{02}^2 p, \tilde{Q} - \Delta_{p, \tilde{Q}}^2 = M^2,$$

where $\chi_{\Delta}, \chi_p, \chi_{\tilde{Q}}$ are the corresponding rapidities, and $V(V_{\lambda_{\tilde{Q}}}, p) = V_{\lambda_p}^{-1} V_{\lambda_{\tilde{Q}}} V_{\Delta p, \tilde{Q}}$ is Wigner's rotation matrix. Hence

follows

$$(35) \quad \frac{\Delta_0^{k', m', \lambda_p}}{m'^2 \Delta_{p, \tilde{Q}}^0} \approx \frac{2M \Delta_0^{k', m', \lambda_{\tilde{Q}}}}{m'^2 \Delta_{p, \tilde{Q}}^0},$$

and the square of 4-momentum transfer of the system $t = (p - \tilde{Q})^2 = -\tilde{Q}^2$ is connected with $\Delta_{p, \tilde{Q}}$ by expression

$$(36) \quad \tilde{Q}^2 = -t = -2M^2 + 2M \sqrt{M^2 + \Delta_{p, \tilde{Q}}^2} + 2M^2 (\cosh \chi_{\Delta} - 1). \quad (36)$$

Therefore, the components $F_{(\pm)}(t)$ of the elastic form factor in (32) and (33) can be considered as functions of the invariant variable $\Delta_{P, Q}^2$, which is the square of the momentum-transfer vector in the Lobachevsky space. With allowance for Eqs. (34), (35) and (36), they are convolutions of the wave functions in this space. It follows that, by employing the Shapiro transformation in (7), the addition theorem^{a)} for relativistic plane waves (4) in the form

$$\int d\omega_n \xi \left(\Delta_{k', m' \lambda_Q} \right) \left(- \right) \left(\frac{m'}{M} \Delta_{P, Q}, \mathbf{r} \right) = \int d\omega_n \xi \left(\Delta_{k', m' \lambda_Q}, \mathbf{r} \right) \xi_* \left(\frac{m'}{M} \Delta_{P, Q}, \mathbf{r} \right),$$

the completeness condition in (5), Eq. (6), and the fact that the free Hamiltonian (14) is a Hermitian operator, one can recast expressions (32) and (33) into the form of relativistic Fourier image of covariant waves RQP-functions in the configuration space.

a) Here, the applicability of the addition theorem follows from the fact that, in the case of $J = 0$, the RQP wave function $\psi^M(\mathbf{r})$ is independent of the direction of the vector \mathbf{r} .

representation^{a)}:

$$F^{(+)}(\hat{Q}_2) \approx \left\{ \frac{m' M (4M_2 + \hat{Q}_2)}{2\mu(z_1+z_2)(2M_2+\hat{Q}_2)} + \frac{m'^2 (4M_2 + \hat{Q}_2)(2M_2+\hat{Q}_2)}{2M_3(z_1-z_2)(m_2^1-m_2^2)} \left(\frac{m'}{2\mu} \right)_3 \right\} \times \left\{ \int d\mathbf{r} \xi_* \left(\frac{m'}{M} \Delta_{P, \hat{Q}_2} \right) \text{Re} [\psi_M^* (\mathbf{r}) \widehat{H}_0 \psi_M (\mathbf{r})] \right\} \times \int d\mathbf{r} \xi_* \left(\frac{m'}{M} \Delta_{P, \hat{Q}_2} \right) \text{Re} \left[\widehat{H}_2 \psi_M (\mathbf{r}) \right] \quad (37)$$

$$F^{(-)}(\hat{Q}_2) \approx \left\{ \frac{m' M \hat{Q}_2}{2\mu(z_1+z_2)(2M_2+\hat{Q}_2)} + \frac{m'^2 \hat{Q}_2 (2M_2 + \hat{Q}_2)}{2M_3(z_1-z_2)(m_2^1-m_2^2)} \left(\frac{m'}{2\mu} \right)_3 \right\} \times \left\{ \int d\mathbf{r} \xi_* \left(\frac{m'}{M} \Delta_{P, \hat{Q}_2} \right) \text{Im} [\psi_M (\mathbf{r}) \widehat{H}_0 \psi_M (\mathbf{r})] + \frac{2M_4 (m'^2 - 4\mu^2)}{m'^4 (2M_2 + \hat{Q}_2)^2} \times \int d\mathbf{r} \xi_* \left(\frac{m'}{M} \Delta_{P, \hat{Q}_2} \right) \text{Im} \left[\widehat{H}_2 \psi_M (\mathbf{r}) \right] \right\} \quad (38)$$

^{a)} A similar expression for the case of two particles of equal mass was obtained in [N. B. Skachkov, I. L. Solovtsov, *Teor. Mat. Fiz.* **43**, 330 (1980)].

We note that, if the RQP wave function $\psi^M(\mathbf{r})$ is a real-valued function of the variable r and corresponds to a real-valued quasipotential $V(r)$, then, according to Eq. (9), the quantity $\psi^M(\mathbf{r}) \left(\widehat{H}_0 \psi^M(\mathbf{r}) \right)^*$ is also real-valued. It follows that, in this case and at equal masses ($m_1 = m_2 = m$), the transverse component $F^{(-)}(t)$ of the elastic form factor vanishes.

In the case where the radial wave function $\varphi_0(r, \chi_n)$ corresponding to a real-valued quasipotential $V(r)$ is real-valued, the quantities $\varphi_0^*(r, \chi_n) \widehat{H}_{0, \ell=0}^r \varphi_0(r, \chi_n)$ and $\left(\widehat{H}_{0, \ell=0}^r \varphi_0(r, \chi_n) \right)^* \widehat{H}_{0, \ell=0}^r \varphi_0(r, \chi_n)$ are also real-valued. It follows that, in this case, the transverse component $F^{(-)}(t)$ of the elastic form factor for the s -wave state ($\ell = 0$) vanishes even for unequal masses ($m_1 \neq m_2$).

For the case where the composite system is the s -wave state ($\ell = 0$), integration with respect to the angular variables in expression (37) yields

$$F_{\ell=0}^{(+)}(\hat{Q}_2) = \left\{ \frac{8\pi\mu(z_1+z_2)(2M_2+\hat{Q}_2)}{8\pi M_3(z_1-z_2)(m_2^1-m_2)} + \frac{m'M(4M_2+\hat{Q}_2)}{m'^2(4M_2+\hat{Q}_2)(2M_2+\hat{Q}_2)} \right\} \times \left(\frac{2\mu}{m'} \right)^3 \times$$

$$\times \frac{\chi_\Delta \sinh \chi_\Delta}{\chi_\Delta} \left\{ \int_0^\infty dr \frac{r \sin(rm'\chi_\Delta)}{\sin(rm'\chi_\Delta)} \operatorname{Re} \left[\frac{r}{\varphi_0^*(r, \chi_n)} \widehat{H}_{0, \ell=0} \right] + \right.$$

$$\left. + \frac{2M^4(m'^2-4\mu^2)}{m'^4(2M_2+\hat{Q}_2)^2} \int_0^\infty dr \frac{r \sin(rm'\chi_\Delta)}{\sin(rm'\chi_\Delta)} \operatorname{Re} \left[\widehat{H}_{0, \ell=0} \left(\frac{r}{\varphi_0(r, \chi_n)} \right) \widehat{H}_{0, \ell=0}^* \left(\frac{r}{\varphi_0(r, \chi_n)} \right) \right] \right\},$$

(39)

where we have used the expansion in (10) for the wave function $\psi_M(\mathbf{r})$ and the expansion for the relativistic plane waves $\xi(\mathbf{p}', \mathbf{r})$ in the form

$$\xi(\mathbf{p}', \mathbf{r}) = \sum_{\ell=0}^{\infty} (2\ell+1) i^\ell p_\ell(r, \cosh \chi_{p'}) P_\ell \left(\frac{\mathbf{p}' \cdot \mathbf{r}}{p' r} \right).$$

We also have

$$\xi_* \left(\frac{m'}{M} \Delta_{p, \varrho, \mathbf{r}} \right) \Big|_{\ell=0} = p_0(rm', \cosh \chi_\Delta) = \frac{\sin(rm' \chi_\Delta)}{\sin m' \sinh \chi_\Delta}$$

where the function

$$p_\ell(r, \cosh \chi_{p'}) = \sqrt{\frac{\pi}{(-1)^{\ell+1}} \frac{2 \sinh \chi_{p'} p}{(-1)^{\ell+1} P_{-1/2-\ell}^{-1/2+i p}(\cosh \chi_{p'})}, p=rm',$$

is a solution of Eq. (6). Here, one refers to

$(-d)^{(\ell+1)} = {}_2F_1(\ell+1, 1+id)/\Gamma(id)$ as the generalized power-law

function, $\Gamma(z)$ being the Euler gamma function. The rapidity χ_Δ is related to the square of the momentum-transfer vector, $\Delta_{p, \varrho}^2$, and the square of the 4-momentum transfer, $\hat{Q}^2 = -t$, in the

Lobachevsky space by Eqs. (36). Further, the rapidity χ_n

corresponds to the bound-state level n the energy

$$M = M_n = (m'^2/\mu) \cosh \chi_n.$$

4. Root-mean-square radius and form factor for Coulomb interaction

The expression for invariant root-mean-square radius $\langle r_0^2 \rangle$ of composite system has the form

$$\langle r_0^2 \rangle = \frac{6\partial F_{\ell=0}^{(+)}(t)/\partial t|_{t=0}}{1} + \frac{F_{\ell=0}^{(+)}(0)}{M_2}$$

$$+ \frac{\left(\frac{m'}{M}\right) \int_0^\infty dr r^2 \left(r^2 - \frac{2m'^2}{3}\right) \left(R_1 + \frac{m'^2 - 4\mu^2}{2m'^4} R_2\right)}{\int_0^\infty dr r^2 \left(R_1 + \frac{m'^2 - 4\mu^2}{2m'^4} R_2\right)}$$

$$+ \frac{3(m'^2 - 4\mu^2) \int_0^\infty dr r^2 R_2}{3(z_1 - z_2)(m_2^1 - m_2^2)(2\mu/m')^2} + \frac{m'^2 M_2 \int_0^\infty dr r^2 \left(R_1 + \frac{m'^2 - 4\mu^2}{2m'^4} R_2\right)}{m'^2 M_2 \left[z_1 + z_2 + \frac{2m'^2}{(z_1 - z_2)(m_2^1 - m_2^2)(2\mu/m')^2} \right]}$$

$$R_1 = \text{Re} \left[\frac{\varphi_0^*(r, \chi)}{\widehat{H}_{0, \ell=0}^p} \right], \quad R_2 = \text{Re} \left[\frac{\varphi_0(r, \chi)}{\widehat{H}_{0, \ell=0}^p} \right], \quad \widehat{H}_{0, \ell=0}^p = \frac{\varphi_0(r, \chi)}{\varphi_0(r, \chi)}$$

Thus, it is necessary to consider the composite particle in question as a dipole where the wave function for the *s-wave state* of this particle describes not all structure of the composite particle, but only the region which be upon distances that larger its of the *Compton wavelength* $1/M$. The root-mean-square radius of the composite system includes not only the central sphere radius $r_0 = 1/M$, where the relative motion of the quarks forming this system proceeds, but also terms generated by the difference in the masses of the quarks and in their coupling constants. At $m_1 = m_2$, these terms vanish.

As example, we consider the form factor of meson in the case of the Coulomb field of attraction between quarks:

$$V(r) = -\frac{\alpha}{r}, \alpha > 0. \quad (40)$$

Solution of RQP-equation (17) with interaction (40) for the s -state and ground level $n = 0$ of bound state with the energy M_0 has the form [O. P. Solovtsova, Yu. D. Chernichenko, Theor. Math. Phys. **166**, 194 (2011); Yu. D. Chernichenko, *Relativistic quasipotential approach in the scattering problems*. (Izdat. centr. UO GTU im. P. O. Sukhogo, Gornel, 2011), 237 p.]:

$$\varphi_0(r, ik_0) = N_{0,0}(k_0) \exp \left[-rm'k_0 + \frac{i\tilde{\alpha}k_0}{2 \sin k_0} \right], \quad (41)$$

where

$$N_{0,0}^2(k_0) = \frac{\pi}{m'k_0^3}, k_0 = \arcsin \frac{\tilde{\alpha}}{2} = \arcsin \frac{m'}{\mu\alpha} = \arccos \frac{\mu M_0}{m'^2}. \quad (42)$$

The form factor (39) for the ground-state level of the bound *s*-state is

$$F_{\ell=0, n=0}^{(+)}(Q_2) = \frac{16\mu\kappa_0^3 \sin \kappa_0 (2M_0^2 + Q_2)}{M_0(4M_0^2 + Q_2) \chi_\Delta \sinh \chi_\Delta} \left[z_1 + z_2 + \right. \quad (43)$$

$$+ \frac{2(z_1 - z_2) M_0^4 (m_1^2 - m_2^2) (2\mu)^2 (m')^2}{1} \left. \left\{ \frac{1 + (2\kappa_0/\chi_\Delta)^2}{4\kappa_0} + \frac{\chi_2^\Delta \tan \kappa_0 (1 + (2\kappa_0/\chi_\Delta)^2)^2}{4\kappa_0} \right. \right.$$

$$+ \frac{4\pi M_0^4 (m'^2 - 4\mu^2) \chi_\Delta \sin 2\kappa_0}{m'^2 (2M_0^2 + Q_2)^2} \left[1 - \frac{\pi}{2} \arctan \frac{\chi_\Delta}{2\kappa_0} + \right.$$

$$\left. \left. + \frac{3}{4\kappa_0} + \frac{\pi \chi_\Delta \tan \kappa_0 (1 + (2\kappa_0/\chi_\Delta)^2)}{3} + \frac{\pi \chi_3^\Delta \tan^2 \kappa_0 (1 + (2\kappa_0/\chi_\Delta)^2)^2}{4\kappa_0} \right] \right\}.$$

For large Q_2 the rapidity behaves as $\chi_\Delta \approx \ln(Q_2/M_0^2)$ and,

consequently, the leading behavior of form factor (43) gives as

$$F_{\ell=0, n=0}^{(+)}(Q_2) \approx 8(z_1 + z_2) \left(\frac{2\mu}{m'} \right)^2 \frac{\kappa_0^3 \tan \kappa_0}{(Q/M_0)^2 \ln(Q/M_0)^2}, \quad (44)$$

i.e. either as in [N. B. Skachkov, I. L. Solovtsov, Theor. Math. Phys. 43, 494 (1980)].

Such behavior of the form factor $t = -Q^2 \gg 1$ differs from

prediction of the nonrelativistic model based on the Coulomb potential, which gives the dipole decrease: $F_\pi \sim t^{-2}$. However, the nonrelativistic result contradicts the prediction of the dimensional quark counting rules [V. A. Matveev, R. M. Muradyan, A. N. Tavkhelidze, Lett. Nuovo Cimento **5**, 907 (1973)], which gives $F_\pi \sim t^{-1}$.

5. Conclusions

For the case of a vector current, new covariant expressions have been found for the components of the elastic form factor for a bound system of two relativistic spinless particles of arbitrary mass m_1, m_2 as functions of the invariant variable $\Delta_{p, q}^2$, which is the square of the momentum-transfer vector in the Lobachevsky space. For this purpose, we have employed the RQP approach based on the covariant Hamiltonian formulation of quantum field theory, implementing this approach via going over to the three-dimensional relativistic configuration representation for a system of two relativistic spinless particles that have arbitrary masses [V. G. Kadyshevsky, M. D. Mateev, and R. M. Mir-Kasimov, *Yad. Fiz.* **11**, 692 (1970); V. G. Kadyshevsky, R. M. Mir-Kasimov, and N. B. Skachkov, *Fiz. Elem. Chastits At. Yadra* **2**, 635 (1972)]. In this approach, the invariant relativistic relative coordinate r is conjugated to the rapidity $m'\chi_\Delta$, and it is the distance in the Lobachevsky space.

It has been shown that expressions (32) and (33) for the form-factor components are convolutions of the RQP wave functions in the space of Lobachevsky angular momenta. This makes it possible to express them in terms of relativistic Fourier transforms of covariant RQP wave functions in the configuration representation. It has also been found that, for a real-valued RQP wave function $\psi^M(\mathbf{r})$ corresponding to a real-valued quasipotential $V(r)$ and in the case of equal masses ($m_1 = m_2 = m$), the transverse component $F_{(-)}^{(l)}(t)$ of the elastic form factor vanishes. Under the same realvaluedness conditions for the *s-wave state* ($l = 0$) of the radial wave function $\varphi_0(r, \chi_n)$ and the quasipotential, the transverse component $F_{(-)}^{l=0}(t)$ of the elastic form factor also vanishes even in the case of unequal masses ($m_1 \neq m_2$).

The application of the three-dimensional relativistic configuration representation for a system of two relativistic particles whose masses are arbitrary made it possible to find that the wave function for the *s-wave state* of this particle describes not all structure of the composite particle, but only the region which be upon distances that larger its of *the Compton wavelength* $1/M$.

The present analysis has revealed that a dominant contribution to the structure of a composite system from the central sphere of radius $1/M$ is proportional to $\chi_\Delta / \sinh \chi_\Delta$, and that the correction terms correspond to the dipole contribution associated with the difference in the masses of the particles constituting this system and in their coupling constants. In the nonrelativistic limit, this relativistic geometric factor tends to unity, while the correction terms in expression (39) at $m_1 = m_2$ vanish.

For the example of an attractive Coulomb field, we have obtained expression (43) for the longitudinal component of the form factor for a system of two relativistic particles that have arbitrary masses. It has been found that the covariant RQP wave function for an attractive Coulomb field at high values of the square of the momentum transfer, $t = -Q^2$, leads to a form-factor behavior that is consistent with the predictions of dimensional quark counting – specifically, to a decrease in proportion to t^{-1} .

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